

**1 Eksamensoppgave 1**

Solve the linear system of equations:

$$3x - 2y + z = 7$$

$$x + y + 2z = 4$$

$$x - y - z = 0$$

Answer:  $x = \square$ ,  $y = \square$ ,  $z = \square$ .

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Maximum marks: 1

**2 Eksamensoppgave 2**

A linear system of equations has augmented matrix with reduced row echelon form:

$$\left[ \begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ 0 & 0 & 0 & a \\ 0 & 0 & 0 & 0 \end{array} \right]$$

How many solutions exist for this system?

**Select one alternative:**

- Infinitely many.
- None.
- Exactly one.
- It depends on the value of  $a$ .

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Maximum marks: 1

**3 Eksamensoppgave 3**

Let

$$A = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & 6 \\ 0 & 3 & 6 \end{bmatrix}.$$

For which vector  $\vec{b}$  does the linear system  $A\vec{x} = \vec{b}$  have a solution?

Select one alternative:

$\vec{b} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$

$\vec{b} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$

$\vec{b} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$

$\vec{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

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Maximum marks: 1

4 **Eksamensoppgave 4 og 5**

$$\text{Let } A = \begin{bmatrix} 1 & 0 & a \\ 0 & 2 & 0 \\ a & 0 & -2 \end{bmatrix}, \text{ with } a \in \mathbb{C}.$$

4) What is the value of  $\det A$ ?

Select one alternative:

- $-4 - 2a^2$
- $-4 + a^2$
- $-4$
- $-4 - 2a$

5) For which values of  $a \in \mathbb{C}$  is  $A$  not invertible?

Select one alternative:

- None
- $2i$  and  $i$
- $a = \pm\sqrt{2}i$
- $a = \pm\sqrt{2}$

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Maximum marks: 2

5 **Eksamensoppgave 6**

Let  $P$  be a parallelogram with vertices  $(1, 2)$ ,  $(3, 1)$ ,  $(-1, -2)$  and  $(-3, -1)$ .

What is the area of  $P$ ? Answer: .

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Maximum marks: 1

## 6 Eksamensoppgave 7 til 10

Let  $A\vec{x} = \vec{b}$  be a linear system, with  $A$  an  $m \times n$ -matrix over  $\mathbb{R}$ . Are the following statements true or false?

7) If the system has at least two solutions, then it has infinitely many solutions.

Select one alternative:

- True
- False

8) If  $m = n$  there is always exactly one solution.

Select one alternative:

- True
- False

9) If two rows of  $A$  are equal, the system has no solution.

Select one alternative:

- True
- False

10) If  $m > n$ , there always exists at least one solution.

Select one alternative:

- True
- False

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Maximum marks: 4

7 **Eksamensoppgave 11 og 12**

$$\text{Let } A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}.$$

11) What are the eigenvalues of  $A$ ?

Select one alternative:

- 2 and 1
- 2
- 2 and 1 and 0
- 1
- 2 and  $\frac{1}{2}$

12) For which of the following matrices  $P$  is  $P^{-1}AP$  a diagonal matrix?

Select one alternative:

- $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
- $P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$
- $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$
- $P = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$

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Maximum marks: 2

8 **Eksamensoppgave 13**

Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be the linear map that reflects vectors across the  $x$ -axis. What are the eigenvalues of  $T$ ?

Select one alternative:

- 1 and 0
- 1 and 0 and  $-1$
- $\pm 1$
- $T$  has no eigenvalues
- $\pm i$

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Maximum marks: 1

9 **Eksamensoppgave 14**

Let  $A = \begin{bmatrix} a & 0 & b \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$ , with  $a, b \in \mathbb{R}$ . Which one of the following statements is true?

Select one alternative:

- $A$  is diagonalizable for all values of  $a$  and  $b \neq 0$
- $A$  is diagonalizable for all values of  $a$  and  $b$
- $A$  is not diagonalizable for any value of  $a$  and  $b$
- $A$  is diagonalizable for all values of  $a$  and  $b = 0$

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Maximum marks: 1

10 **Eksamensoppgave 15**

Let  $A$  be an invertible matrix with an eigenvalue  $\lambda = 3$ . Which one of the following statements is true?

Select one alternative:

- 3 is an eigenvalue of  $A^{-1}$
- We have no information about the eigenvalues of  $A^{-1}$
- $\frac{1}{3}$  is an eigenvalue of  $A^{-1}$
- $-3$  is an eigenvalue of  $A^{-1}$

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Maximum marks: 1

## 11 Eksamensoppgave 16 til 20

Let  $A$  be a real  $3 \times 3$ -matrix, and let  $B$  be a complex  $3 \times 3$ -matrix. Are the following statements true or false for all such matrices?

16)  $A$  has no complex eigenvalues.

Select one alternative:

- True
- False

17)  $B$  has at least one real eigenvalue.

Select one alternative:

- True
- False

18)  $B$  is diagonalizable.

Select one alternative:

- True
- False

19)  $A$  has at least one real eigenvalue.

Select one alternative:

- True
- False

20) If  $A$  is symmetric, its eigenvalues are real.

Select one alternative:

- True
- False

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Maximum marks: 5

12 **Eksamensoppgave 21 til 23**

Are the following subsets vector spaces?

$$21) \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \mid x = y \right\} \subseteq \mathbb{R}^3$$

Select one alternative:

- Yes
- No

$$22) \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \mid |x| = |y| \right\} \subseteq \mathbb{R}^3$$

Select one alternative:

- Yes
- No

$$23) \{ p(x) = ax^2 + bx + c \in \mathcal{P}_2 \mid p'(x) = 0 \} \subseteq \mathcal{P}_2$$

Select one alternative:

- Yes
- No

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Maximum marks: 3



13 **Eksamensoppgave 24 til 26**

Let  $A$  be a  $3 \times 3$ -matrix and let  $I_3$  be the  $3 \times 3$ -identity matrix. Are the following functions linear maps?

$$24) T : \mathbb{R}^3 \rightarrow \mathbb{R}^3, \quad T(\vec{x}) = A\vec{x} + \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}.$$

Select one alternative:

Yes

No

$$25) T : \mathbb{R}^3 \rightarrow \mathbb{R}^3, \quad T(\vec{x}) = (A^2 + I_3)\vec{x}.$$

Select one alternative:

Yes

No

$$26) T : \mathcal{P}_2 \rightarrow \mathcal{P}_2, \quad T(p(x)) = 2p'(x) + p(x) + 1$$

Select one alternative:

Yes

No

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Maximum marks: 3

14 **Eksamensoppgave 27 til 30**

Let  $V$  be a vector space with  $\dim V = n \geq 2$ . Are the following statements true or false?

27) All sets of  $n - 1$  vectors in  $V$  are linearly independent.

Select one alternative:

- True
- False

28) All sets of  $n + 1$  vectors in  $V$  are linearly dependent.

Select one alternative:

- True
- False

29) There exists no injective linear map  $T : V \rightarrow V$  such that  $T^2 = 0$ .

Select one alternative:

- True
- False

30) A linear map  $T : V \rightarrow V$  that satisfies  $T^2 = T$  must be injective.

Select one alternative:

- True
- False

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Maximum marks: 4

**15 Eksamensoppgave 31 og 32**

Consider the point set  $(-1, -1)$ ,  $(0, 0)$  and  $(1, 2)$  in  $\mathbb{R}^2$ .

31) Which straight line is the best fit?

Select one alternative:

- $y = 3x + 2$
- $y = 2x + 3$
- $y = \frac{1}{2}x + \frac{3}{2}$
- $y = \frac{3}{2}x + \frac{1}{3}$

32) Which second-degree polynomial travels through all points?

Select one alternative:

- $y = \frac{1}{2}x^2 + \frac{3}{2}$
- $y = \frac{3}{2}x^2 + \frac{1}{2}$
- $y = \frac{3}{2}x^2 + \frac{1}{2}x$
- $y = \frac{1}{2}x^2 + \frac{3}{2}x$

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Maximum marks: 2

16 **Eksamensoppgave 33 til 35**

Let  $T : \mathcal{P}_2 \rightarrow \mathcal{P}_2$  be given by  $T(p(x)) = p'(x) + p''(x)$ , and let  $\mathcal{B} = (1, x, x^2)$  be a basis for  $\mathcal{P}_2$ .

33) What is  $[T]_{\mathcal{B}}$  ?

Select one alternative:

$[T]_{\mathcal{B}} = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

$[T]_{\mathcal{B}} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$[T]_{\mathcal{B}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$[T]_{\mathcal{B}} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & 2 & 0 \end{bmatrix}$

34) What is  $\dim \text{Null } T$ ? Answer: .

35) Which statement is true?

Select one alternative:

- 0 is the only eigenvalue of  $T$
- $T$  has no eigenvalues
- 1 and 2 are eigenvalues of  $T$
- 0, 1 and 2 are eigenvalues of  $T$

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Maximum marks: 3

17 **Eksamensoppgave 36**

A solution of  $y'' - 4y' + 4y = 0$  satisfies  $y(0) = 0$  and  $y'(0) = 1$ .

What is  $y(1)$  ? Answer: . (Round the answer to three decimal places.)

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Maximum marks: 1

18 **Eksamensoppgave 37**

Let  $\vec{y}$  be a solution of  $\vec{y}' = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \vec{y}$ . What is  $\lim_{t \rightarrow \infty} \vec{y}(t)$ ?

Select one alternative:

- $\begin{bmatrix} \infty \\ \infty \end{bmatrix}$
- $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$
- $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$
- $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

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Maximum marks: 1

19 **Eksamensoppgave 38**

Let  $z = a + bi$  be a complex number. There is no  $w$  such that  $zw = 1$  if:

Select one alternative:

- $z \neq 0$
- $a = 0$  and  $b = 0$
- $b = 0$
- $a = 0$

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Maximum marks: 1

20 **Eksamensoppgave 39**

To multiply a complex number by  $z = \frac{\sqrt{3}}{2} + \frac{1}{2}i$  is a rotation. A rotation by how many radians?

Answer:  radians. (Round the answer to three decimal places.)

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Maximum marks: 1

## 21 Eksamensoppgave 40

Dissociate Professor Moreon Bulde has to work from home this spring. He is bored, but enjoys sporadic fiddling with numbers. In addition he loves to eat eggs, and always initiates his workdays with either runny eggs, fried eggs, or chocolate chip eggs. The day after having eaten chocolate chip eggs it is equally likely that he eats runny eggs or fried eggs, and he NEVER eats chocolate chip eggs two days in a row. The day after runny or fried eggs, he repeats his meal with 50% probability, or else he eats chocolate chip eggs. In the long run, what does he eat most frequently?

**Select one alternative:**

- The meals are equally frequent.
- Mostly fried eggs.
- Mostly chocolate chip eggs.
- Eggs Benedict.

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Maximum marks: 1