Eksempel:

La $f(t) = \sin(t)$ for $0 \leq t \leq \pi$. Finn Fourierrekke-utviklingen til $f(t)$ ved å bruke *odde* og *like* utvidelse av funksjonen.

Løsning:

Den odde utviklingen av $f(t)$ er

$$\tilde{f}(t) = \begin{cases} \sin(t) & , 0 \leq t \leq \pi \\ -\sin(-t) & , -\pi < t < 0 \\ f(t+2\pi) & , \text{ellers} \end{cases}$$
$$= \begin{cases} \sin(t) & , 0 \leq t \leq \pi \\ \sin(t) & , -\pi < t < 0 \\ f(t+2\pi) & , \text{ellers} \end{cases}$$

Altså $f(t) = \sin(t)$.

Fourierrekka til den odde utviklingen av $f(t)$ er trivielt $f(t) = \sin(t)$.

Den like utviklinga av $f(t)$ er

$$\tilde{f}(t) = \begin{cases} \sin(t) & , 0 \leq t \leq \pi \\ \sin(-t) & , -\pi < t < 0 \\ f(t+2\pi) & , \text{ellers} \end{cases}$$

Siden dette er ei like utvikling, vet vi at koeffisientene $b_k = 0$ for alle k.

Altså trenger vi bare å bestemme a_0 og a_k for $k \geq 1$.

Vi finner a_0:

$$
\begin{aligned}
a_0 &= \frac{1}{2\pi}\int_{-\pi}^{\pi}\tilde{f}(t)dt = \frac{1}{\pi}\int_0^{\pi}\tilde{f}(t)dt \\
&= \frac{1}{\pi}\int_0^{\pi} f(t)dt = \frac{1}{\pi}\int_0^{\pi}\sin(t)dt \\
&= \Big|_0^{\pi} -\frac{1}{\pi}\cos(t) = -\frac{1}{\pi}\left[\cos(\pi)-\cos(0)\right] = \frac{2}{\pi}.
\end{aligned}
$$

Vi finner a_k:

$$
\begin{aligned}
a_k &= \frac{1}{\pi}\int_{-\pi}^{\pi}\tilde{f}(t)\cos(kt)dt = \frac{2}{\pi}\int_0^{\pi}\tilde{f}(t)\cos(kt)dt \\
&= \frac{2}{\pi}\int_0^{\pi} f(t)\cos(kt)dt = \frac{2}{\pi}\int_0^{\pi}\sin(t)\cos(kt)dt.
\end{aligned}
$$

For $k = 1$ gir dette

$$
a_1 = \frac{2}{\pi}\int_0^{\pi}\sin(t)\cos(kt)dt = \frac{1}{\pi}\int_0^{\pi}\sin(2t)dt = 0.
$$

Nå har vi at

$$
\sin(x)\cos(y) = \frac{1}{2}\left[\sin(x+y)+\sin(x-y)\right],
$$

slik at

$$\begin{aligned}
a_k &= \frac{2}{\pi}\int_0^\pi \sin(t)\cos(kt)dt \\
&= \frac{1}{\pi}\int_0^\pi \sin([1+k]t) + \sin([1-k]t)dt \\
&= \frac{1}{\pi}\int_0^\pi \sin([k+1]t) - \sin([k-1]t)dt \\
&= \frac{1}{\pi}\Bigg|_0^\pi - \frac{\cos([k+1]t)}{k+1} + \frac{\cos([k-1]t)}{k-1} \\
&= \frac{1}{\pi}\left(\frac{-\cos([k+1]\pi)+1}{k+1} + \frac{\cos([k-1]\pi)-1}{k-1}\right)
\end{aligned}$$

For k odde, vil

$$\begin{aligned}
\cos([k+1]\pi) &= 1 \\
\cos([k-1]\pi) &= 1
\end{aligned}$$

For k like, vil

$$\begin{aligned}
\cos([k+1]\pi) &= -1 \\
\cos([k-1]\pi) &= -1
\end{aligned}$$

Dette gir

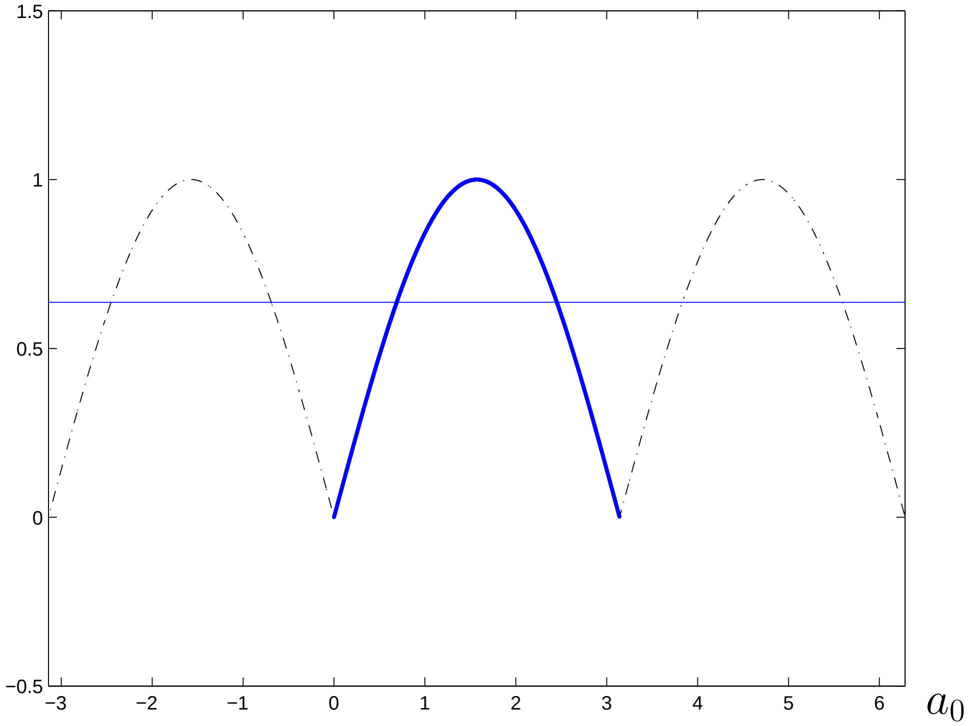
$$a_k \begin{cases} 0 & , k \text{ odde} \\ \frac{2}{\pi}\left(\frac{1}{k+1} - \frac{1}{k-1}\right) & , k \text{ like} \end{cases}$$

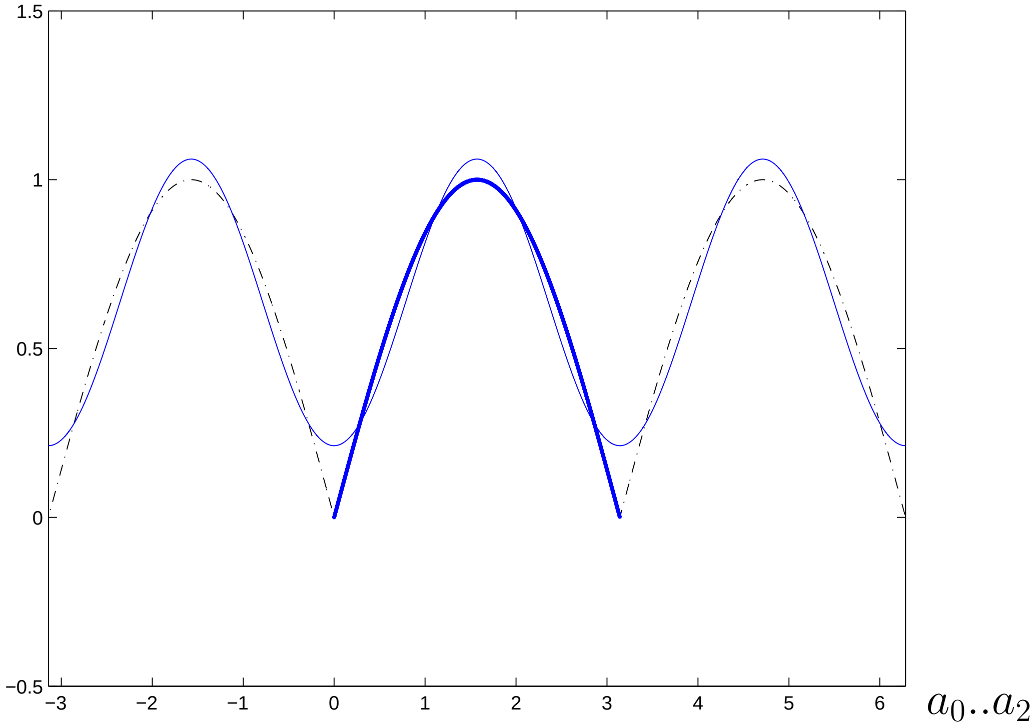
Eller

$$a_k \begin{cases} 0 & , k \text{ odde} \\ -\frac{4}{\pi(k+1)(k-1)} & , k \text{ like} \end{cases}$$

Den like utviklingen av $f(t)$ blir altså

$$\underline{f(t) = \frac{2}{\pi} - \frac{4}{\pi}\left(\frac{\cos(2t)}{1\cdot 3} + \frac{\cos(4t)}{3\cdot 5} + \frac{\cos(6t)}{5\cdot 7} + \dots\right)}$$





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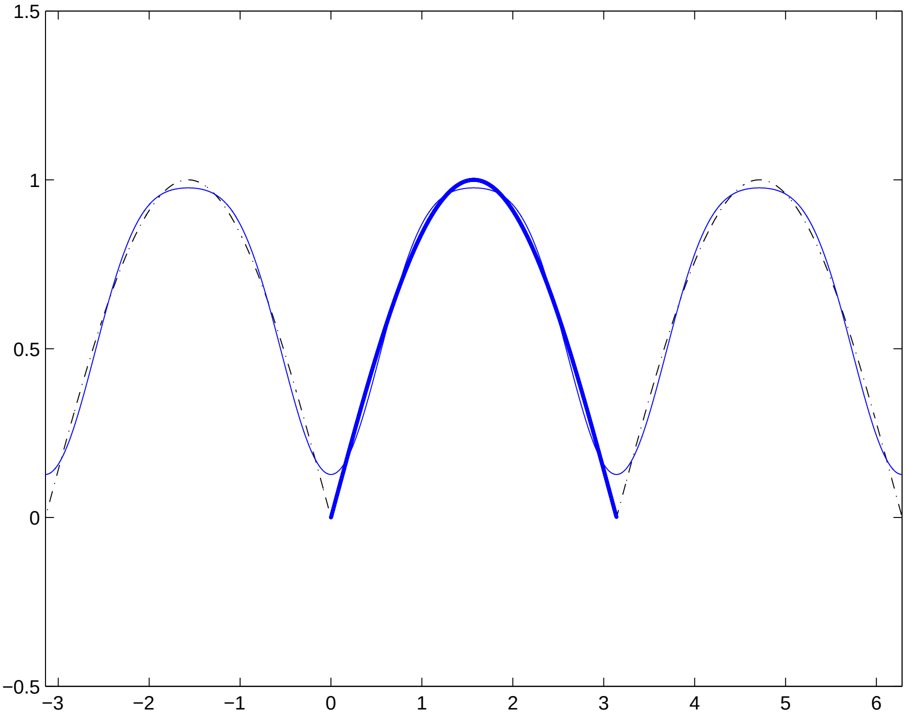
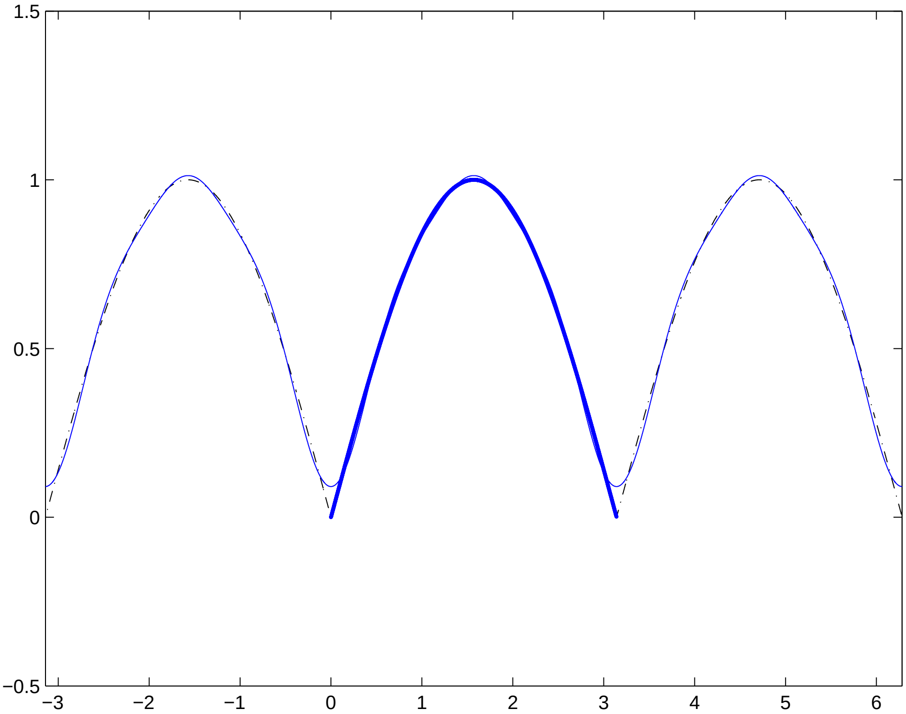


$a_0..a_4$



$a_0..a_6$

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