Løsning: $u(x,t) = \varphi(x+ct) + \psi(x-ct)$

Initialbetingelser: $u(x,0) = f(x)$, $u_t(x,0) = g(x)$

$$u(x,0) = \varphi(x) + \psi(x) = f(x) \qquad (1)$$

$$u_t(x,0) = c\varphi'(x) + \psi'(x)(-c) = g(x) \qquad (2)$$

$$(1) \qquad \varphi(x) + \psi(x) = f(x) \qquad (1)$$

$$\int_{x_0}^{x} \frac{(2)}{c}\,dx: \quad \varphi(x) - \varphi(x_0) - \psi(x) + \psi(x_0) = \int_{x_0}^{x} g(x)\,dx \qquad (2')$$

$$(1)+(2'): \qquad \varphi(x) = \frac{1}{2}\left\{ f(x) + \int_{x_0}^{x} g(x)\,dx + \varphi(x_0) - \psi(x_0) \right\}$$

$$(1)-(2'): \qquad \psi(x) = \frac{1}{2}\left\{ f(x) - \int_{x_0}^{x} g(x)\,dx - \varphi(x_0) + \psi(x_0) \right\}$$

$$\begin{aligned} u(x,t) &= \varphi(x+ct) + \psi(x-ct) \\ &= \frac{1}{2}\left\{ f(x+ct) + f(x-ct) + \int_{x_0}^{x+ct} g(x)\,dx - \int_{x_0}^{x-ct} g(x)\,dx \right\} \\ &= \frac{1}{2}\left\{ f(x+ct) + f(x-ct) + \int_{x-ct}^{x+ct} g(x)\,dx \right\} \end{aligned}$$