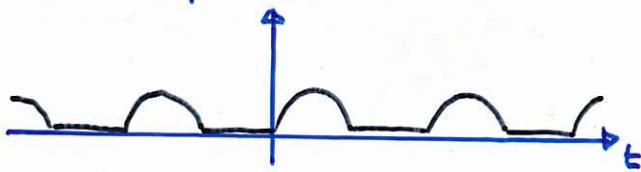


Eksempel



$$u(t) = \begin{cases} 0 & \text{for } -\frac{\pi}{\omega} < t < 0 \\ \sin \omega t & \text{for } 0 < t < \frac{\pi}{\omega} \end{cases}$$

Find Fourierrekken $a_0 + \sum (a_n \cos n\omega t + b_n \sin n\omega t)$

$$a_0 = \frac{1}{2L} \int_{-L}^L u(t) dt = \frac{\omega}{2\pi} \int_{-\pi/\omega}^0 0 dt + \int_0^{\pi/\omega} \sin \omega t dt = \frac{1}{\pi}$$

$$a_n = \frac{1}{L} \int_{-L}^L u(t) \cos n\omega t dt = \frac{\omega}{\pi} \int_0^{\pi/\omega} \sin \omega t \cos n\omega t dt$$

~~$$= \frac{\omega}{2\pi} \left[\frac{\sin(1+n)\omega t}{(1+n)\omega} - \frac{\sin(1-n)\omega t}{(1-n)\omega} \right]_0^{\pi/\omega}$$~~

$$= \frac{\omega}{2\pi} \left\{ \int_0^{\pi/\omega} \sin(1+n)\omega t dt \quad \text{①} \quad + \quad \int_0^{\pi/\omega} \sin(1-n)\omega t dt \quad \text{②} \right\}$$

①:
$$\begin{cases} 0 & n \text{ odde} \\ \frac{2}{(1+n)\omega} & n \text{ like} \end{cases}$$

②:
$$\begin{cases} 0 & n \text{ odde} \\ \frac{2}{(1-n)\omega} & n \text{ like} \end{cases}$$

$$a_n = \begin{cases} 0 & n \text{ odde} \\ \frac{2}{(1-n^2)\pi} & n \text{ like} \end{cases}$$

$$b_n = \begin{cases} \frac{1}{2} & \text{for } n=1 \\ 0 & \text{ellers} \end{cases}$$

Derfor:

$$\begin{aligned} u(t) &= \frac{1}{\pi} + \frac{1}{2} \sin \omega t + \frac{2}{\pi(1-2^2)} \cos 2\omega t + \frac{2}{\pi(1-4^2)} \cos 4\omega t + \dots \\ &= \frac{1}{\pi} + \frac{1}{2} \sin \omega t - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos 2n\omega t}{4n^2-1} \end{aligned}$$