

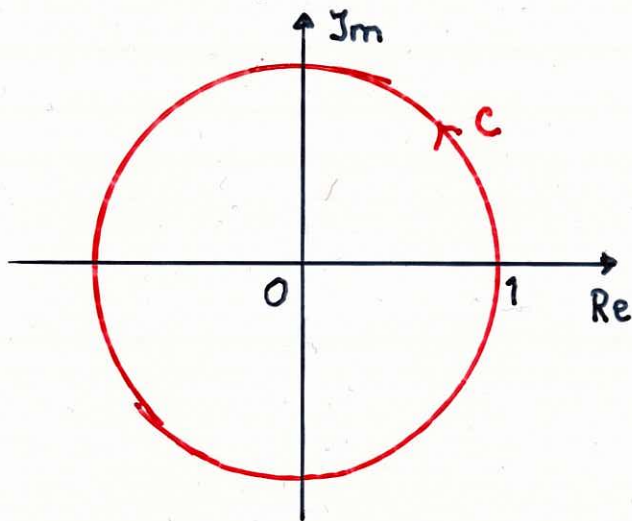
Oppgave 15.4.2

$$J = \int_0^{2\pi} \frac{d\theta}{25 - 24 \cos \theta} = ?$$

$$dz = i e^{i\theta} d\theta = iz d\theta$$

$$d\theta = \frac{dz}{iz}$$

Substitusjon: $z = e^{i\theta}$



Integrasjonsvei C: $|z| = 1$

$$\begin{aligned} 25 - 24 \cos \theta &= 25 - 24 \frac{e^{i\theta} + e^{-i\theta}}{2} \\ &= 25 - 12 \left(z + \frac{1}{z} \right) \end{aligned}$$

$$J = \oint_C \frac{dz}{iz (25 - 12z - 12/z)} = \frac{1}{i} \oint_C \frac{dz}{25z - 12z^2 - 12}$$

Faktoriserer nevner: $12z^2 - 25z + 12 = 0 \Rightarrow z = \frac{3}{4}, z = \frac{4}{3}$

$$\begin{aligned} J &= \oint_C \frac{1}{i} \cdot \frac{1}{-12} \cdot \frac{dz}{(z - \frac{3}{4})(z - \frac{4}{3})} = -\frac{1}{12i} \cdot 2\pi i \cdot \operatorname{Res}_{z = \frac{3}{4}} \frac{1}{(z - \frac{3}{4})(z - \frac{4}{3})} \\ &= -\frac{\pi}{6} \cdot \left[\frac{1}{z - \frac{4}{3}} \right]_{z = \frac{3}{4}} = -\frac{\pi}{6} \cdot \frac{1}{\frac{3}{4} - \frac{4}{3}} = -\frac{\pi}{6} \cdot \frac{12}{-7} = \underline{\underline{\frac{2\pi}{7}}} \end{aligned}$$