

## Oppgave 10.6.2

$$f(x) = |x| \text{ for } -\pi < x < \pi \quad \text{periodisk, } 2\pi$$

Tilnærm  $f$  med best mulig trig. polynom av grad  $N$   
Beregn kvadratisk feil. ( $N = 1, 2, 3, 4, 5$ )

Jevn funksjon  $\Rightarrow$  Fourier cosinus polynom ( $b_n = 0$ )

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |x| dx = \frac{2}{2\pi} \int_0^{\pi} x dx = \frac{1}{\pi} \cdot \frac{\pi^2}{2} = \frac{\pi}{2}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} |x| \cos nx dx = \frac{2}{\pi} \int_0^{\pi} x \cos nx dx$$

$$= \frac{2}{\pi} \left[ \frac{1}{n^2} \cos nx + \frac{x}{n} \sin nx \right]_0^{\pi} = \frac{2}{\pi} \left( \frac{1}{n^2} \cos n\pi - \frac{1}{n^2} \right)$$

$$= \frac{2}{\pi n^2} \left( (-1)^n - 1 \right) = \begin{cases} 0 & \text{for } n \text{ jevn} \\ -\frac{4}{\pi n^2} & \text{for } n \text{ odde} \end{cases}$$

Fourier-rekken:  $f(x) = \frac{\pi}{2} - \frac{4}{\pi 1^2} \cos x - \frac{4}{\pi 3^2} \cos 3x - \frac{4}{\pi 5^2} \cos 5x - \dots$

$N=1,2$ : Feil =  $\int_{-\pi}^{\pi} f^2 dx - 2a_0^2 \pi - \cancel{2} a_1^2 \pi$

$$= \int_{-\pi}^{\pi} x^2 dx - 2\pi \cdot \frac{\pi^2}{4} - \frac{16}{\pi^2} \pi = \frac{2\pi^3}{3} - \frac{\pi^3}{2} - \frac{16}{\pi}$$

$N=3,4$  Feil =  $\frac{2\pi^3}{3} - \frac{\pi^3}{2} - \frac{16}{\pi} - \frac{16}{91\pi} \pi$

osv