1 Consider the boundary value problem for the wave equation:

$$u_{tt} = c^2 u_{xx}, \quad t > 0, \quad 0 < x < \pi, \quad u(t,0) = u(t,\pi) = 0. \tag{1}$$

(a) Find all solutions of (1) on the form $u(t,x) = F(t)G(x)$.

(b) Find the solution of (1) that also satisfies the following initial condition

$$u(0,x) = \pi x - x^2, \quad u_t(0,x) = 0, \quad 0 < x < \pi.$$

2 Consider the boundary value problem for the Laplace equation:

$$u_{xx} + u_{yy} = 0, \quad 0 < x < \pi, \quad 0 < y < 2\pi \quad u(0,y) = 0, \quad u_x(\pi,y) = 0. \tag{2}$$

(a) Find all solutions of (2) on the form $u(x,y) = F(x)G(y)$.

(b) Find the solution of (2) that also has the following values on the horizontal sides

$$u(x,0) = u(x,2\pi) = \sin\frac{3x}{2} + 4\sin\frac{7x}{2} + -5\sin\frac{11x}{2}.$$

3 Solve exercise 12.3.5 in Kreyszig.