D. Approks. m. trig. polynom ($p = 2\pi$)

Ønsker:

$$f(x) \simeq P_k(x) := A_0 + \sum_{n=1}^{k} (A_n \cos nx + B_n \sin nx), \quad x \in [-\pi, \pi]$$

trig. polynom orden k (for $A_0, A_n, B_n \in \mathbb{R}$)

Obs 3:

F. coeff.

$$S_{f,k}(x) := a_0 + \sum_{n=1}^{k} (a_n \cos nx + b_n \sin nx) \underset{n \text{ stor}}{\simeq} S_f(x) \overset{\text{hvis konv.}}{=} f(x)$$

trig. poly.

$P_k = S_{f,k}$ er trig. poly. m. minst L^2 ("mean square") feil:

$$\| f - P_k \|^2 := \int_{-\pi}^{\pi} | f(x) - P_k(x) |^2 dx \quad :$$

Tm. 2: Anta f stk. vis kont. på $[-\pi, \pi]$.

a) $\| f - S_{f,k} \|^2 \leq \| f - P_k \|^2$

for alle trig. poly. P_k (alle A_0, A_n, B_n)

b) $\| f - S_{f,k} \|^2 = \int_{-\pi}^{\pi} f^2 dx - \pi \left[2a_0^2 + \sum_{n=1}^{k} (a_n^2 + b_n^2) \right]$

Eks. 4:

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases}, \quad f(x+2\pi) = f(x)$$

$$S_f(x) = \frac{4}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \dots\right)$$

Finn k og $S_{f,k}$ s.a. $\| f - S_{f,k} \|^2 < \frac{1}{2}$:

$$k = 0: \quad \| f - S_{f,0} \|^2 = \underline{2\pi}, \qquad S_{f,0} = 0$$

7.

$$k = 1: \quad \| f - S_{f,1} \|^2 \overset{\text{Tm. 2 b}}{=} \underbrace{\int_{-\pi}^{\pi} f^2 \, dx}_{= 2\pi} - \pi \left(\frac{4}{\pi}\right)^2 \simeq 1{,}2, \qquad S_{f,1} = \frac{4}{\pi}\sin x$$

$$\vdots$$

$$\underline{\underline{k = 5}}: \quad \| f - S_{f,5} \|^2 \overset{\text{Tm. 2b}}{=} \int_{-\pi}^{\pi} f^2 \, dx - \pi \left(\frac{4}{\pi}\right)^2 \left(1^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{5}\right)^2\right) \simeq \underline{0{,}41}$$

$$S_{f,5} = \frac{4}{\pi}\left(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x\right)$$