EXAM IN CALCULUS 4N (TMA4125)
Monday June 06, 2005
Time: 09:00 - 13:00

Hjelpemidler: Simple calculator (HP 30S), Rottmann: matematisk formelsamling

Grades: 27.06.05

Oppgave 1
a) Find the inverse Laplace transform of the function

\[ F(s) = \frac{s + 4}{(s + 2)^2} \]

b) Solve the initial value problem:

\[ y''(t) + 4y'(t) + 4y(t) = 0, \quad t \geq 0, \quad y(0) = 1, \quad y'(0) = 0. \]

c) Solve the integral equation:

\[ y(t) + \int_0^t e^{-(t-\tau)} y(\tau)d\tau = e^{-2t}, \quad t > 0. \]

Oppgave 2
a) Find Fourier series for 2\pi-periodic even function \( f(t) \) such that \( f(x) = \frac{\pi}{2} - x, \ 0 < x < \pi. \)
b) Find all solutions of the form \( u(x, t) = X(x)T(t) \) for the problem:

(1) \( \frac{\partial^2 u}{\partial x^2} - 2u - \frac{\partial u}{\partial t} = 0, \ 0 < x < \pi, \ t > 0. \)

(2) \( u_x(0, t) = 0, \ u_x(\pi, t) = 0, \ t > 0. \)

c) Find the solution to the problem (1), (2) in part b.) which also satisfies the initial condition

\[ u(x, 0) = \frac{x}{\pi} - x, \ 0 < x < \pi. \]

**Oppgave 3**

Find complex Fourier transform of the function \( f(x) = e^{-|x|} \) and then find the value of the integral

\[ \int_{-\infty}^{\infty} \frac{\cos \omega}{1 + \omega^2} d\omega \]

**Oppgave 4**

- a) Find a polynomial of the smallest possible degree which solves the interpolation problem

| \( x_k \) | -2 | -1 | 0 | 1 | 2 |
| \( p(x_k) \) | 6 | 0 | 0 | 0 | 15 |

- b) Let \( p(x) \) be the polynomial from part a). Using the Simpson method with step 1 evaluate the integral \( \int_{-2}^{2} p(x) dx \).
Oppgave 5

We are solving partial differential equation

\[ u_t = u_{xx}, \quad -1 \leq x \leq 1, \quad t \geq 0 \]

\[ u(x,0) = 1 - x^2, \]

\[ u(-1,t) = 0, \quad u(1,t) = 0, \quad t \geq 0. \]

Let \( \kappa = 0.5, h = 0.5. \) Using the Crank-Nicolson method write down the system of linear equations for the values

\[ u_{11} \approx u(-0.5, 0.5), \quad u_{21} \approx u(0, 0.5), \quad u_{31} \approx u(0.5, 0.5). \]