Numerical formulas

- Let $p(x)$ be the polynomial of degree $\leq n$ which coincides with $f(x)$ at points $x_i, i = 0, 1, \dots, n$. Under the assumption that x and all the nodes x_j lie in the interval $[a, b]$, we have
$$f(x) - p(x) = \frac{1}{(n+1)!} f^{(n+1)}(\xi) \prod_{i=0}^{n} (x - x_i).$$
- Newton's divided difference interpolation formula $p(x)$ of degree $\leq n$:
$$\begin{aligned} p(x) = f[x_0] + (x - x_0) f[x_0, x_1] + (x - x_0)(x - x_1) f[x_0, x_1, x_2] \\ + \cdots + (x - x_0)(x - x_1) \dots (x - x_{n-1}) f[x_0, \dots, x_n] \end{aligned}$$
- Simpson's rule of integration:
$$\int_{x_0}^{x_2} f(x)\, \mathrm{d}x \approx \frac{h}{3}(f_0 + 4f_1 + f_2)$$
Error bounded by $h^4 \frac{b-a}{180} \max_{a \leq x \leq b} |f^{(4)}(x)|$.
- Newton's method for solving a system of nonlinear equations $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ is given by the scheme
$$\begin{aligned} J^{(k)} \cdot \Delta \mathbf{x}^{(k)} &= -\mathbf{f}(\mathbf{x}^{(k)}) \\ \mathbf{x}^{(k+1)} &= \mathbf{x}^{(k)} + \Delta \mathbf{x}^{(k)}. \end{aligned}$$
- Iteration methods for solving systems of linear equations $A\mathbf{x} = \mathbf{b}$ when $A_{i,i} = 1$:
$$\text{Jacobi:} \quad \mathbf{x}^{(m+1)} = \mathbf{b} - (A - I)\mathbf{x}^{(m)}$$
$$\text{Gauss–Seidel:} \quad \mathbf{x}^{(m+1)} = \mathbf{b} - L\mathbf{x}^{(m+1)} - U\mathbf{x}^{(m)}$$
Strict diagonal dominance of A is a sufficient convergence criterion for both.
- Butcher tables for Runge–Kutta methods, where
$$\mathbf{y}_{n+1} = \mathbf{y}_n + \sum_{i=1}^{s} b_i \mathbf{k}_i, \qquad \mathbf{k}_i = h\mathbf{f}(x_n + c_i h, \mathbf{y}_n + \sum_{j=1}^{s} a_{i,j} \mathbf{k}_j):$$

(Forward) Euler:

$$\begin{array}{c|c} 0 & 0 \\ \hline & 1 \end{array}$$

Backward Euler:

$$\begin{array}{c|c} 1 & 1 \\ \hline & 1 \end{array}$$

Heun/improved Euler:

$$\begin{array}{c|cc} 0 & 0 & 0 \\ 1 & 1 & 0 \\ \hline & 1/2 & 1/2 \end{array}$$

RK4:

$$\begin{array}{c|cccc} 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ \hline & 1/6 & 1/3 & 1/3 & 1/6 \end{array}$$

- Discrete Fourier transform:
$$\hat{f}_n = \sum_{k=0}^{N-1} f_k e^{-2\pi i n k/N}$$

Table of some Laplace transforms

$f(t)$	$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t)\ \mathrm{d}t$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
$t^n \ (n = 0, 1, 2, ...)$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$e^{at}\cos \omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
$e^{at}\sin \omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$
$\delta(t-a)$	e^{-as}

Table of some Fourier transforms

$f(x)$	$\hat{f}(\omega) = \mathcal{F}\{f(x)\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\omega x}\ \mathrm{d}x$
$g(x) = f(ax)$	$\hat{g}(\omega) = \frac{1}{a}\hat{f}\left(\frac{\omega}{a}\right)$
e^{-ax^2}	$\frac{1}{\sqrt{2a}} e^{-\frac{\omega^2}{4a}}$
$e^{-a\|x\|}$	$\sqrt{\frac{2}{\pi}} \frac{a}{\omega^2+a^2}$
$u(x) - u(x-a)$	$\frac{1}{\sqrt{2\pi}}\left(\frac{\sin a\omega}{\omega} - i\frac{1-\cos a\omega}{\omega}\right)$
$u(x)e^{-x}$	$\frac{1}{\sqrt{2\pi}}\left(\frac{1}{1+\omega^2} - i\frac{\omega}{1+\omega^2}\right)$