Norwegian University of Science and Technology
Department of Mathematical Sciences

TMA4125 Calculus 4N
Spring 2006

Solutions to exercise set 10

17.2.7 We are to find the smallest positive solution to $x = \tan x$. To ensure convergence of fixed-point iteration of $x = g(x)$ we need $|g'(x)| \le q < 1$. Since $\frac{\mathrm{d}}{\mathrm{d}x} \tan x = \frac{1}{\cos^2 x}$, convergence is not ensured.

Since $\tan x = \tan(x - \pi)$, we are able to use $x = \pi + \arctan x$ instead. Then we have $\frac{\mathrm{d}}{\mathrm{d}x}(\pi + \arctan x) = \frac{1}{1+x^2} \le q < 1$. This is bounded below 1 around 4 (where we know the solution has to be, by looking at the graph of $\tan x$). π is in other words added to move us away from origo.

Iteration then yields, starting at $x = 0$:

$$\begin{aligned}
x_0 &= 0 \\
x_1 &= 3.14159265358979 \\
x_1 &= 4.40421990926870 \\
x_1 &= 4.48911945509409 \\
x_1 &= 4.49320682642241 \\
x_1 &= 4.49339989522730 \\
x_1 &= 4.49340900664090 \\
x_1 &= 4.49340943661351
\end{aligned}$$

17.2.11 We want to find $x = \sqrt[3]{7}$. Newtons method then use $f(x) = x^3 - 7 = 0$, and we get

$$x_{n+1} = x_n - \frac{x_n^3 - 7}{3x_n^2}$$

$$\begin{aligned}
x_0 &= 2 \\
x_1 &= 1.91666666666667 \\
x_1 &= 1.91293845830708 \\
x_1 &= 1.91293118280006
\end{aligned}$$

17.3.6 $p_2(x) = L_0(x)f_0 + L_1(x)f_1 + L_2(x)f_2$. In example 2 this is:

$$p_2(x) = -0.00523x^2 + 0.205016x + 0.77595$$

Then we calculate:

x	$p_2(x)$	$\ln(x)$	error
9.4	2.2409776	2.2407096	0.000268
10	2.3031100	2.3025850	0.000525
10.5	2.3520105	2.3513752	0.000635
11.5	2.4419665	2.4423470	0.000380
12	2.4830220	2.4849066	0.001884

As expected, the error is growing as we move away from our points.

17.3.13 Five points gives a polynomial of degree four, with five coefficients. These may be zero, so we may axpect a polynomial of degree four or less.

You may use your method of choice, any valid method will give the same polynomial (see page 849 for the subject of uniqueness). Since our points are equally spaced, we could also use formula (14) or (18). Here I will use the standard Newton Divided Difference method of interpolation.

x	y				
1	5				
		13			
2	18		3		
		19		0	
3	37		3		0
		25		0	
4	62		3		
		31			
5	93				

Then our polynom is:

$$p(x) = 5 + 13(x - 1) + 3(x - 1)(x - 2)$$

which reduces to

$$p(x) = 3x^2 + 4x - 2$$

It is always recommended to check that this polynomial goes through our points. That is: $p(x) = y$ for all points (x, y)

17.3.14 First we fill in the table:

x	$\ln(x)$		
9	2.1972		
		0.1082	
9.5	2.2513		-0.0052335
		0.097733	
11	2.3979		

Then our polynom is:

$$p(x) = 2.1972 + 0.1082(x - 9) - 0.0052335(x - 9)(x - 9.5)$$

which reduces to

$$p(x) = 0.7759 + 0.2050x - 0.0052335x^2$$

Then we calculate:

x	p(x)	ln(x)
9.4	2.2405	2.2407096
10	2.3026	2.3025850
10.5	2.3514	2.3513752
11.5	2.4413	2.4423470
12	2.4823	2.4849066