Norwegian University of Science and Technology
Department of Mathematical Sciences

# TMA4125 Matematikk 4N
# Spring 2006

**Solutions to exercise set 6**

**10.8.16** We are to represent the function $f(x) = \pi - x$ for $0 < x < \pi$ and $f(x) = 0$ for $x > \pi$ as a Fourier Sine Integral:

$$f(x) = \int_0^\infty B(w) \sin wx \, dw$$

where

$$B(w) = \frac{2}{\pi} \int_0^\infty f(v) \sin wv \, dv$$

$$\begin{aligned}
B(w) &= \frac{2}{\pi} \int_0^\infty f(v) \sin wv \, dv \\
&= 2 \int_0^\pi \sin wv \, dv - \frac{2}{\pi} \int_0^\pi v \sin wv \, dv \\
&= \left| -\frac{2}{w} \cos wv \right|_0^\pi - \left( \left| \frac{2}{\pi w^2} \sin wv - \frac{2v}{\pi w} \cos wv \right|_0^\pi \right) \\
&= -\frac{2}{w} \cos w\pi + \frac{2}{w} - \frac{2}{\pi w^2} \sin w\pi + \frac{2}{w} \cos w\pi \\
&= \frac{2}{w} - \frac{2}{\pi w^2} \sin w\pi
\end{aligned}$$

Then we have

$$f(x) = \int_0^\infty \left( \frac{2}{w} - \frac{2}{\pi w^2} \sin w\pi \right) \sin wx \, dw$$

*Comment.* The denominator in the expression for $B$ vanishes at the point $w = 0$. But you can collect all in one fraction and then check that $B$ is bounded near the origin (use for example l'Hopitalle rule, or Taylor expansion.)

$$\lim_{w \to 0} \left\{ \left( \frac{2}{w} - \frac{2}{\pi w^2} \sin w\pi \right) \sin wx \right\}$$

etc