Separation of variables

- Look for elementary solutions of the form u(x,t) = X(x)T(t) of the equation you have (do not think about boundary or initial conditions). Obtain two ordinary differential equations which contain an unknown constant k, say;
- Use initial/boundary conditions in order to complete one of these ordinary equations with extra conditions. From this equation and these conditions find the sequence $\{k_n\}$ of possible values of the constant and also the corresponding sequence of elementary solutions $\{X_n(x)T_n(t)\}$.
- Look for the solution of the whole problem in the form

$$u(x,t) = \sum_{n} c_n X_n(x) T_n(t),$$

the coefficients $\{c_n\}$ are to be found in order to meet the remaining boundary/initial condition.