TMA4123 / TMA4125
Matematikk 4M/N
Vår 2007

Norwegian University of Science
and Technology
Institutt for matematiske fag

Exercise set 1

Kreyzsig (8th ed): 10.1.8 Sketch or plot the following function $f(x)$, which is assumed to be periodic with period 2π and, for $-\pi < x < \pi$, is given by the formula

$$f(x) = x^2.$$

Kreyzsig (8th ed): 10.1.16 Sketch or plot the following function $f(x)$, which is assumed to be periodic with period 2π and, for $-\pi < x < \pi$, is given by the formula

$$f(x) = \begin{cases} x & \text{if} \quad -\pi < x < 0 \\ \pi - x & \text{if} \quad 0 < x < \pi \end{cases}.$$

Kreyzsig (8th ed): 10.2.4 Showing the details of your work, find the Fourier series of the function $f(x)$, which is assumed to have the period 2π, where $f(x)$ equals
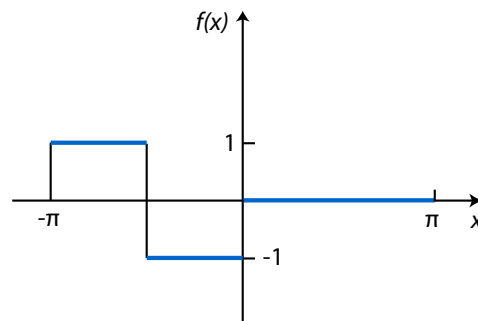
Kreyzsig (8th ed): 10.2.6 Showing the details of your work, find the Fourier series of the function $f(x)$, which is assumed to have the period 2π, where $f(x)$ equals

$$f(x) = x \quad (0 < x < 2\pi).$$

Kreyzsig (8th ed): 10.2.13 Showing the details of your work, find the Fourier series of the function $f(x)$, which is assumed to have the period 2π, where $f(x)$ equals

$$f(x) = \begin{cases} 1 & \text{if} \quad -\pi/2 < x < \pi/2 \\ -1 & \text{if} \quad \pi/2 < x < 3\pi/2 \end{cases}.$$

Kreyzsig (8th ed): 10.2.15 Showing the details of your work, find the Fourier series of the function $f(x)$, which is assumed to have the period 2π, where $f(x)$ equals

$$f(x) = \begin{cases} x & \text{if} \quad -\pi/2 < x < \pi/2 \\ 0 & \text{if} \quad \pi/2 < x < 3\pi/2 \end{cases}.$$

7 Prove the orthogonality relations.

$$\int_{-\pi}^{\pi} \cos mx \cos nx\, dx = 0, \quad \text{where } m \text{ and } n \text{ are integers and } m \neq n$$

$$\int_{-\pi}^{\pi} \sin mx \sin nx\, dx = 0, \quad \text{where } m \text{ and } n \text{ are integers and } m \neq n$$

$$\int_{-\pi}^{\pi} \cos mx \sin nx\, dx = 0, \quad \text{for any integers } m \text{ and } n$$

8 Find the sum of the series
$$\sum_{l=1}^{\infty} \frac{(-1)^l}{2l+1}.$$

Hint: Consider the Fourier series of the 2π-periodic function f defined as

$$f(x) = \begin{cases} 0 & \text{if} \quad -\pi \leq x \leq 0 \\ \pi - x & \text{if} \quad 0 < x < \pi \end{cases}.$$

9 Find the Fourier series for the function

$$f(x) = (\sin x + \cos^2 x)^2.$$