TMA4123 / TMA4125
Matematikk 4M/N
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**Exercise set 5**

**1** Find the function $e_T(t)$ such that

$$\hat{e}_T(u) = \begin{cases} 1 & |u| < T \\ 0 & \text{otherwise} \end{cases}$$

**Kreyzsig (8th ed): 10.10.8** Find the Fourier transform of the following function $f(x)$ (without using Table III, Sec. 10.11). Show the details of your work.

$$f(x) = \begin{cases} e^{x} & \text{if} \quad x < 0 \\ e^{-x} & \text{if} \quad x > 0 \end{cases}$$

**Kreyzsig (8th ed): 10.10.15** Solve problem 7 by convolution. (Problem 7 was done in exercise set 4.)

**Kreyzsig (8th ed): 11.1.8** Verify that $u = e^{-9t} \cos \omega x$ is a solution of the heat equation (with suitable $c$). Sketch or plot a figure of the solution as a surface in space.

**Kreyzsig (8th ed): 11.1.14b** Verify that $u = 1/\sqrt{x^2 + y^2 + z^2}$ satisfies Laplace's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0.$$

**Kreyzsig (8th ed): 11.1.23** Verify that $u(x, y) = a \ln(x^2 + y^2) + b$ satisfies Laplace's equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

and determine $a$ and $b$ so that $u$ satisfies the boundary conditions $u = 0$ on the circle $x^2 + y^2 = 1$ and $u = 3$ on the circle $x^2 + y^2 = 4$. Sketch a figure of the surface represented by this function.

**Kreyzsig (8th ed): 11.3.3** Find $u(x, t)$ of the string of length $L = \pi$ when $c^2 = 1$, the initial velocity is zero, and the initial deflection is $k(\sin x - \frac{1}{2} \sin 2x)$.

$\boxed{\text{Kreyzsig (8th ed): 11.3.6}}$ Find $u(x,t)$ of the string of length $L = \pi$ when $c^2 = 1$, the initial velocity is zero, and the initial deflection is