Norwegian University of Science and Technology
Institutt for matematiske fag

Exercise set 6

Exercises from Kreyszig (8th ed):

$\boxed{1}$ *Exercise 11.5.3*

Find the temperature $u(x,t)$ in a bar of silver (length 10 cm, constant cross section of area $1\,\mathrm{cm}^2$, density $10.6\,\mathrm{g/cm}^3$, thermal conductivity $1.04\,\mathrm{cal/(cm\,sec\,°C)}$, specific heat $0.056\,\mathrm{cal/(g\,°C)}$) that is perfectly insulated laterally, whose ends are kept at temperature 0°C and whose initial temperature in (°C) is $f(x)$, where

$$f(x) = \sin 0.1\pi x.$$

$\boxed{2}$ *Exercise 11.5.7*

Different temperatures at the ends What is the limit $u_I(x)$ of the temperature of the bar in the text as $t \to \infty$ if the ends are kept at $u(0,t) = U_1 = \text{const}$ and $u(L,t) = U_2 = \text{const}$?

$\boxed{3}$ *Exercise 11.5.15*

The **boundary condition of heat transfer**

$$-u_x(\pi,t) = k\,[u(\pi,t) - u_0]$$

applies when a bar of length π with $c = 1$ is laterally insulated, the left end $x = 0$ is kept at 0°C, and at the right end heat is flowing into air of constant temperature u_0. Let $k = 1$ for simplicity, and $u_0 = 0$. Show that a solution is $u(x,t) = \sin px e^{-p^2 t}$, where p is a solution of $\tan p\pi = -p$. Show graphically that this equation has infinitely many positive solutions $p_1, p_2, p_3, \ldots$, where $p_n > n - 1/2$ and $\lim_{n\to\infty}(p_n - n + 1/2) = 0$.

$\boxed{4}$ *Exercise 11.5.17*

(Heat flow in a plate) The faces of a thin square copper plate (Figure 1, where $a = 24$) are perfectly insulated. The upper side is kept at 20°C and the other sides are kept at 0°C. Find the steady-state temperature $u(x,y)$ in the plate. (Solve the Laplace equation.)

Exam problem

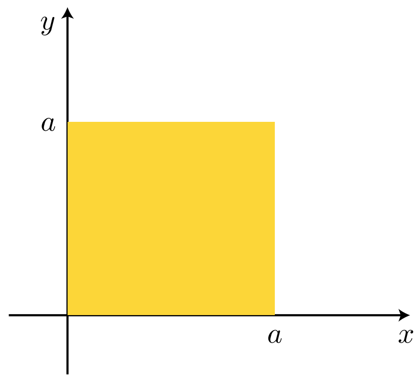

Figure 1: Square plate

$\boxed{5}$ *SIF5013 mai 2000 oppgave 2*

a) Funksjonen $f(x)$ er definert ved

$$f(x) = \begin{cases} 1 & \text{for} \quad 0 < x < \pi/2, \\ 0 & \text{for} \quad \pi/2 < x < \pi. \end{cases}$$

Tegn 2 perioder (fra -2π til 2π) av grafen til den jevne, 2π-periodiske utvidelsen av $f(x)$, og finn Fouriercosinusrekka til $f(x)$.

b) Gitt en partiell differensialligning (1) med randbetingelser (2):

$$\frac{\partial^2 u}{\partial x^2} = u + \frac{\partial u}{\partial t} \tag{1}$$

$$u_x(0,t) = 0 \quad \text{og} \quad u_x(\pi,t) = 0 \quad \text{for } t > 0. \tag{2}$$

Finn alle løsninger av (1) på formen $u(x,t) = F(x)G(t)$ som oppfyller (2).

c) Finn en formell løsning av (1) som, i tillegg til (2), oppfyller initialbetingelsen

$$u(x,0) = f(x) \quad \text{for } 0 \le x \le \pi, \tag{3}$$

der $f(x)$ er funksjonen gitt i **a)**.
Finn også en løsning av (1) som oppfyller (2) og initialbetingelsen

$$u(x,0) = 2\cos x \cos 2x. \tag{4}$$