Numerisk integrasjon

Vi bruker de 4 numeriske integrasjonsmetodene vi har sett til å beregne

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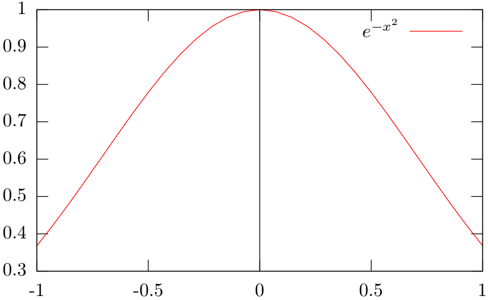

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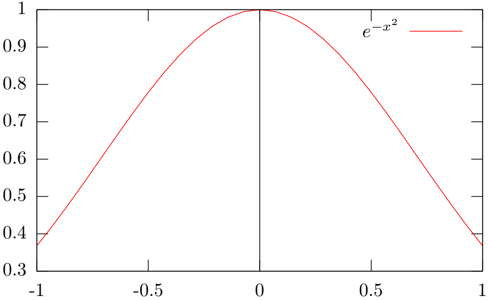
$$\int_{-1}^{1} e^{-x^2}\, dx = 1.493648265$$



Rektangulær metode

Vi har

$$S(f) = h \sum_{i=0}^{n} f\left(\frac{x_i + x_{i+1}}{2}\right)$$

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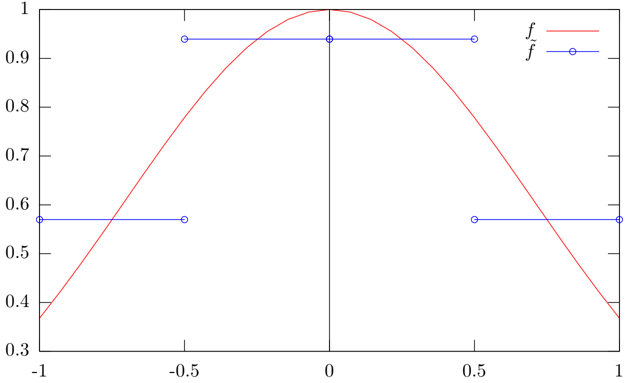

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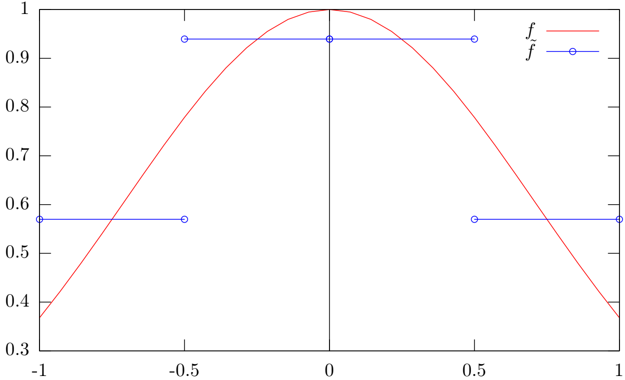
$$S(f) = h\sum_{i=0}^{n} f\left(\frac{x_i + x_{i+1}}{2}\right) = \int_{-1}^{1} \tilde{f}(x)\,dx$$
$$= \frac{2}{4}\cdot\left(f(-0.75) + f(-0.25) + f(0.25) + f(0.75)\right)$$



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$$\begin{aligned} S(f) &= h \sum_{i=0}^{n} f\left(\frac{x_i + x_{i+1}}{2}\right) = \int_{-1}^{1} \tilde{f}(x)\, dx \\ &= \frac{2}{4} \cdot (f(-0.75) + f(-0.25) + f(0.25) + f(0.75)) \\ &= 1.509195887 \end{aligned}$$

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Totalfeilen:

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\epsilon = \int_{-1}^{1} f(x)\,dx - S(f)
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&= -0.0155476219
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$$S(f) = h \sum_{i=0}^{n-1} \left(\frac{f(x_i) + f(x_{i+1})}{2} \right)$$

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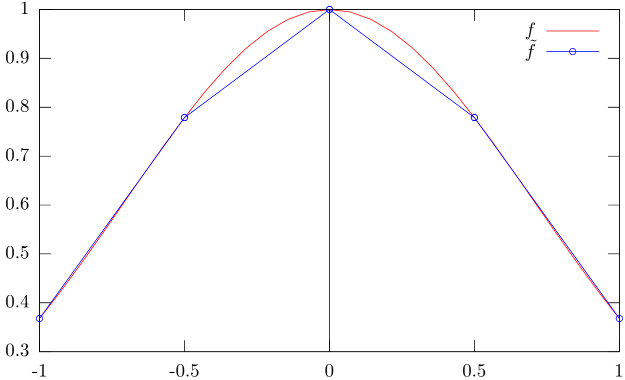

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$$S(f) = h\sum_{i=0}^{n-1}\left(\frac{f(x_i)+f(x_{i+1})}{2}\right) = \int_{-1}^{1}\tilde{f}(x)\,dx$$

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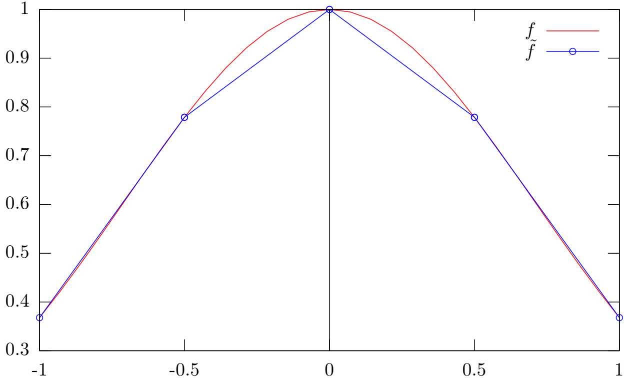

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Totalfeilen:

$$\epsilon = \int_{-1}^{1} f(x)\,dx - S(f)$$
$$= 0.0309077619$$

Simpsons metode

Vi har

$$S(f) = h \sum_{i=0}^{m-1} \left(\frac{1}{3} f(x_{2i}) + \frac{4}{3} f(x_{2i+1}) + \frac{1}{3} f(x_{2i+2}) \right)$$

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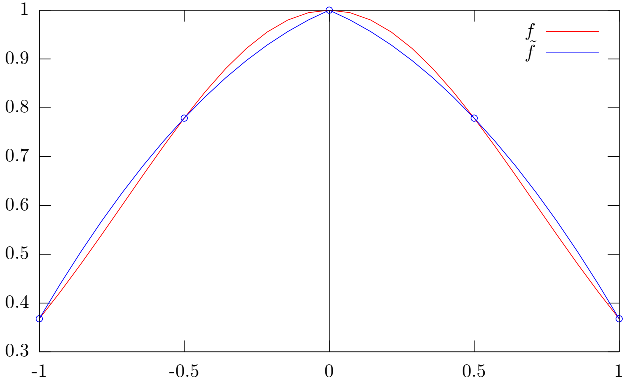

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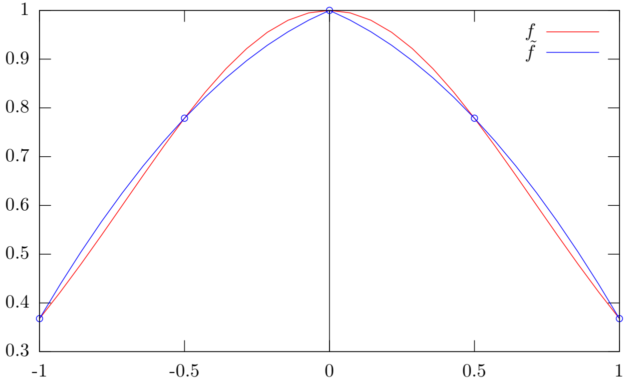

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Totalfeilen:

$$
\begin{aligned}
\epsilon &= \int_{-1}^{1} f(x)\, dx - S(f) \\
&= -7.12592195 \cdot 10^{-4}
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$$

Degree of precision

Definition

degree of precision til en integrasjonsmetode er tallet N slik at metoden integrerer eksakt for alle polynomene opp til graden N.

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for alle polynomene f slik at

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Man har

$$\text{orden til metoden} = \text{DP}+1$$

metode	Feilestimat	DP
Rektangulær	$\lvert\epsilon\rvert \leq Kh^1$	0
Trapesformet	$\lvert\epsilon\rvert \leq Kh^2$	1
Simpsons	$\lvert\epsilon\rvert \leq Kh^4$	3

Gauss integrasjonsmetode (1)

Vi bruker 5 noder:

x_j	A_j
-0.9061798459	0.2369268851
-0.5384693101	0.4786286705
0	0.5688888889
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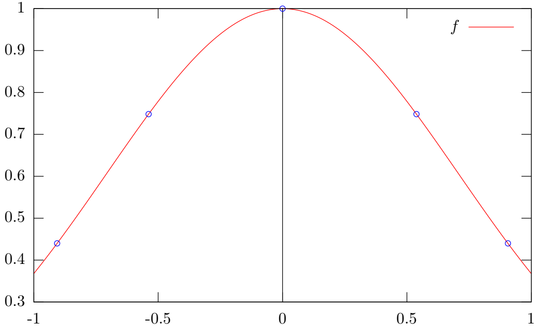

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$$S(f) = \sum_{j=1}^{n} A_j f(x_j)$$

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$$S(f) = \sum_{j=1}^{n} A_j f(x_j)$$
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$$= 1.49366392077$$

Feil:

$$\epsilon = \int_{-1}^{1} f(x)\,dx - S(f)$$
$$= -1.565514514e - 5$$

Sammenliging mellom metodene

metode	$\lvert\epsilon\rvert$
Rektangulær	0.015
Trapesformet	0.031
Simpson	0.00071
Gauss	0.000016

Gauss integrasjonsmetode (2)

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