

Formler i numerikk

- La $p(x)$ være et polynom av grad $\leq n$ som interpolerer $f(x)$ i punktene $x_i, i = 0, 1, \dots, n$. Forutsatt at x og alle nodene ligger i intervallet $[a, b]$, så gjelder

$$f(x) - p(x) = \frac{1}{(n+1)!} f^{(n+1)}(\xi) \prod_{i=0}^n (x - x_i)$$

Hvis nodene er jevnt fordelt (inkludert endepunktene), og $|f^{(n+1)}(x)| \leq M$, da gjelder

$$|f(x) - p(x)| \leq \frac{1}{4(n+1)!} M \left(\frac{b-a}{n} \right)^{n+1}$$

- Numerisk derivasjon:

$$\begin{aligned} f'(x) &= \frac{1}{h} (f(x+h) - f(x)) + \frac{1}{2} h f''(\xi) \\ f'(x) &= \frac{1}{h} (f(x) - f(x-h)) - \frac{1}{2} h f''(\xi) \\ f''(x) &= \frac{1}{h^2} (f(x+h) - 2f(x) + f(x-h)) - \frac{1}{12} h^2 f^{(4)}(\xi) \end{aligned}$$

- Newtons metode for ligningssystemet $\mathbf{f}(\mathbf{x}) = \mathbf{0}$ er gitt ved

$$\begin{aligned} \mathbf{J}^{(k)} \cdot \Delta \mathbf{x}^{(k)} &= -\mathbf{f}(\mathbf{x}^{(k)}) \\ \mathbf{x}^{(k+1)} &= \mathbf{x}^{(k)} + \Delta \mathbf{x}^{(k)} \end{aligned}$$

- Iterative teknikker for løsning av et lineært ligningssystem

$$\sum_{j=1}^n a_{ij} x_j = b_i, \quad i = 1, 2, \dots, n$$

$$\begin{aligned} \text{Jacobi :} \quad x_i^{(k+1)} &= \frac{1}{a_{ii}} \left(b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k)} - \sum_{j=i+1}^n a_{ij} x_j^{(k)} \right) \\ \text{Gauss-Seidel :} \quad x_i^{(k+1)} &= \frac{1}{a_{ii}} \left(b_i - \sum_{j=1}^{i-1} a_{ij} x_j^{(k+1)} - \sum_{j=i+1}^n a_{ij} x_j^{(k)} \right) \end{aligned}$$

- En 2. ordens Runge-Kutta metode (Heun) for $\mathbf{y}' = \mathbf{f}(x, \mathbf{y})$:

$$\begin{aligned} \mathbf{K}_1 &= h \mathbf{f}(x_n, \mathbf{y}_n) \\ \mathbf{K}_2 &= h \mathbf{f}(x_n + h, \mathbf{y}_n + \mathbf{K}_1) \\ \mathbf{y}_{n+1} &= \mathbf{y}_n + \frac{1}{2} (\mathbf{K}_1 + \mathbf{K}_2) \end{aligned}$$