TMA4125 Matematikk 4N

Øving 2 - Laplacetransform II

Obligatoriske oppgaver

$\boxed{1}$ Bevis konvolusjonsteoremet for laplacetransform.

$\boxed{2}$ Lag et script som plotter Heavisidefunksjonen på intervallet $[-\pi, \pi]$.

$\boxed{3}$ a) $y'' + 4y' + 5y = \delta(t-1) \quad y(0) = 0,\ y'(0) = 3$

b) $y'' + 5y' + 6y = \delta\left(t - \frac{\pi}{2}\right) + u(t-\pi)\cos t \quad y(0) = 0,\ y'(0) = 0$

c) $ty'' - ty' + y = 1, \quad y(0) = 1,\ y'(0) = 2$

d) $y(t) - \int_0^t y(\tau)(t-\tau)\,\mathrm{d}\tau = 2 - \dfrac{1}{2}t^2$

Anbefalte oppgaver

$\boxed{1}$ Utled formlene for s-skift og t-skift.

$\boxed{2}$ Finn de inverse laplacetransformene

a) $\frac{1}{s^2(s^2+1)}$

b) $\frac{s}{s^2+2s+1}$

c) $\frac{2s}{(s^2+1)^2}$

d) $(s-3)^{-5}$

$\boxed{3}$ Finn laplacetransformene

a) $f(t) = (u(t) - u(t-\pi))\cos t$

b) $f(t) = u(t-a)t^2$, $a > 0$

c) $f(t) = u(t) + 2\sum_{i=1}^{\infty}(-1)^i u(t-ia)$, $a > 0$

$\boxed{4}$ Løs likningene

a) $y'' + y = u(t-\pi) \quad y(0) = 0,\ y'(0) = 0$

b) $y - y \star t = t$.

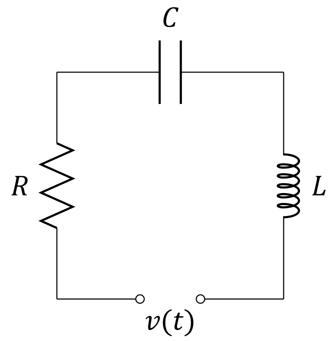
$\boxed{5}$ Strømmen $i(t)$ i kretsen under er gitt ved likningen

$$Li'(t) + Ri(t) + \frac{1}{C}\int_0^t i(\tau)d\tau = v(t). \qquad (1)$$

Anta $i(0) = i'(0) = 0$, $R = 4\ \Omega$, $L = 1$ H, $C = 0.05$ F og

$$v(t) = \begin{cases} 34e^{-t}\text{ V} & \text{if } 0<t<4, \\ 0\text{ V} & \text{ellers.} \end{cases}$$

FInn strømmen i.



Nøtt a) Detail the calculation showing that $\mathcal{L}(t^n)(s) = \frac{\Gamma(n+1)}{s^{n+1}}$, where $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$, $(x > 0)$, is the classical Gamma-function.

b) Show that $\Gamma(x+1) = x\Gamma(x)$ and calculate $\Gamma(\frac{2k+1}{2})$, for k being a non-negative integer.

c) Show that $\Gamma(1/2) = 2\int_0^\infty e^{-p^2}dp$.

d) Prove that $2\int_0^\infty e^{-p^2}dp = \sqrt{\pi}$.

e) Combine part b), c) and d) to calculate the Laplace transform $\mathcal{L}(\frac{2}{\sqrt{\pi}}\int_0^{\sqrt{t}} e^{-p^2}dp)$.

Hint (at least three ways):

1. Expanding the exponential e^{-p^2} by using $e^x = 1 + x + \frac{x^2}{2} + \cdots + \frac{x^n}{n!} + \cdots$;

2. Use definition of Laplace transform and two variables calculus;

3. Compute derivative of $\int_0^{\sqrt{t}} e^{-p^2}dp$ then use $\mathcal{L}(f')(s) = sF(s) - f(0)$.