Norwegian University of Science and Technology
Department for Mathematical Science

TMA4125 Calculus 4N
Spring 2020

Exercise set 7
Wave equation

Mandatory exercises

1 In the following exercise you are asked to work out the details of Section "Løsning for fløyte" from Morten's Lecture Notes on the wave equation:

A standing pressure wave in a flute of length L can be described by the wave equation

$$\partial_t^2 u(x,t) = c^2 \partial_x^2 u(x,t), \quad x \in (0,L), \quad t > 0, \tag{1}$$

together with the Neumann boundary conditions

$$\partial_x u(0,t) = \partial_x u(L,t) = 0, \quad t > 0, \tag{2}$$

and the initial conditions

$$u(x,0) = f(x), \quad \partial_t u(x,0) = g(x), \quad x \in (0,L). \tag{3}$$

Then the solution to the problem (1)–(3) is

$$u(x,t) = A + \sum_{n=1}^{\infty} \Big(A_n \cos c\frac{n\pi}{L} t + B_n \sin c\frac{n\pi}{L} t \Big) \cos \frac{n\pi}{L} x, \tag{4}$$

where A is an arbitrary constant, and

$$A_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi}{L} x \, \mathrm{d}x, \tag{5}$$

and

$$B_n = \frac{2}{cn\pi} \int_0^L g(x) \cos \frac{n\pi}{L} x \, \mathrm{d}x. \tag{6}$$

Similar to our derivation of the solution representation for the heat problem, use the technique of separation of variables and provide a detailed derivation of the solution representation (4)–(6).

2 Use the solution representation derived in Lecture 14 and compute the solution to the wave equation on a bounded interval $I = (0,1)$:

$$\partial_t^2 u(x,t) = 25 \partial_x^2 u(x,t), \quad x \in (0,1), t > 0, \tag{7}$$

together with the Dirichlet boundary conditions

$$u(0,t) = u(1,t) = 0, \quad t > 0, \tag{8}$$

and the initial conditions

$$u(x,0) = x - x^2, \quad \partial_t u(x,0) = 1 - x \quad \text{for } x \in (0,1). \tag{9}$$

3 Use the solution representation derived in Lecture 14 and compute the solution to the wave equation on the entire real line $\mathbb{R}$

$$\partial_t^2 u(x,t) = 25\partial_x^2 u(x,t), \qquad \text{for } x \in \mathbb{R}, t > 0, \tag{10}$$

together with the initial conditions

$$u(x,0) = e^{-x^2}, \quad \partial_t u(x,0) = x, \quad x \in \mathbb{R}. \tag{11}$$

Recommended exercises

4 The vibrations in a flute are solutions to the problem (1)–(2), where $c = 343m/s$ is the speed of sound at atmospheric pressure, and L is the length of the flute. Find all the solutions of the form $F_n(x)G(t)$. What is the deepest frequency a flute of length 50 cm can create?