exercise_05

February 14, 2021

1 Fourier Series

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Date: **Feb 15, 2021** Deadline: **Mar 8, 2021, 12:00**

If you want to have a nicer theme for your jupyter notebook, download the cascade stylesheet file tma4125.css and execute the next cell:

[2]:
```
from IPython.core.display import HTML
def css_styling():
    try:
        with open("tma4125.css", "r") as f:
            styles = f.read()
            return HTML(styles)
    except FileNotFoundError:
        pass #Do nothing

# Comment out next line and execute this cell to restore the default notebook␣
 ↪style
css_styling()
```

[2]:
```
<IPython.core.display.HTML object>
```

1.1 1)

A function is called even if $f(-x) = f(x)$ and it is called odd if $f(-x) = -f(x)$ for all $x \in \mathbf{R}$. Show that if f is even then

$$\int_{-L}^{L} f(x)dx = 2\int_{0}^{L} f(x)dx$$

and if f is odd

$$\int_{-L}^{L} f(x)dx = 0$$

for any $L \in \mathbf{R}$.

2 2)

Let V be the space of polynomials of degree ≤ 2 on the interval $[0, 3]$. Compute the best approximation of $f(x) = \cos(x)$ in V. You may use the results obtained in exercise sheet 3, problem 1.2.

2.1 3)

Consider the function

$$f(x) = \begin{cases} 3 & -\pi \leq x \leq 0 \\ 1 & 0 < x \leq \pi \end{cases}$$

a) Find its Fourier series in complex form.

b) Find its Fourier series in trigonometric form, calculating again the Fourier coefficients by integration.

c) Verify that the two series you obtained in fact coincide, by using the relation between Fourier coefficients in complex and trigonometric form.

2.2 4)

Let $f(x) = x^2$.

a) Find its Fourier series in complex form.

b) Use the relation between Fourier coefficients in complex and trigonometric form to find the trigonometric series of f (do not calculate again the integrals!).

c) Evaluate the sum $\sum_{n=1}^{+\infty} \frac{1}{n^4}$. **Hint:** Use Parseval's equality.

2.3 5)

a) Find the trigonometric Fourier series of $f(x) = 3\sin(4x) + 5\cos(3x)$.

b) (Bonus, optional) Find the trigonometric Fourier series of $\sin(\sqrt{2}x)$.

2.4 6)

Let $g(x) = x + f(x)$, where $f(x)$ is the function defined in Problem 3. Compute the trigonometric Fourier series of g. You may and should reuse computations already done in the lectures.