Norges teknisk–naturvitenskapelige universitet
Institutt for matematiske fag

TMA4125
Vår 2021

Øving 7

Mandatory Exercises

The deadline for handing in solutions is ***Monday 22th of March, 12:00****.*

1 Solve the following initial-value problems by using the Laplace transform

a)

$$y''(t) - 7y(t) = 0, \quad y(0) = e^{-2}, \quad y'(0) = 56.$$

b)

$$y''(t) + 2y'(t) + y(t) = e^{-2t}, \quad y(0) = y'(0) = 0.$$

c)

$$y''(t) + 4y'(t) + 4y(t) = \int_0^t (t-x)^2 \cos(x) dx, \quad y(0) = y'(0) = 0.$$

Hint: Consider the convolution theorem.

d)

$$y'(t) + y(t) = \delta(t-4) + 1, \quad y(0) = 2.$$

2 Compute the inverse Laplace transform of the functions

a)

$$F(s) = \frac{e^{\pi \cdot s}}{e^{\pi}(s-1)}.$$

Hint: Use the first shifting theorem.

b)

$$F(s) = \frac{s^2}{(s+2)(s^2+1)^2}.$$

Hint: Split the fraction.

3 Compute the Laplace transform of

a)

$$f(t) = t^2 \cdot \cosh(t).$$

Hint: Use the first shifting theorem.

b)

$$f(t) = u(t + \frac{\pi}{2}) \sin(t).$$

Hint: Use the second shifting theorem.

c)

$$f(t) = \int_0^t \frac{\sin(\pi \cdot (t-x))\sqrt{8x}}{t-x} dx.$$

Hint: Recall the convolution theorem.

Recommended Exercises

4 Solve the following initial-value problems by using the Laplace transform

a)

$$y''(t) + 5y'(t) + 6y(t) = 0, \quad y(0) = -2, \quad y'(0) = 1.$$

b)

$$y'' - 2y' + 2y = 6e^{-t} \quad y(0) = 0, \quad y'(0) = 1.$$

c)

$$y'' + 5y' + 6y = \delta(t-2), \quad y(0) = 1, \quad y'(0) = -1.$$

d)

$$y'(t) - \int_0^t (t-\tau)y(\tau)d\tau = t, \quad y(0) = 1.$$

5 Use the shift-theorem for Laplace transforms to show that the Laplace transform of the function $f(t) = \cosh(t)\cos(t)$ is

$$F(s) = \frac{1}{2}\left(\frac{s-1}{(s-1)^2+1} + \frac{s+1}{(s+1)^2+1}\right).$$

6 Calculate the inverse Laplace transform of the functions

a)

$$F(s) = \frac{2s}{s^2 - 8s + 6}.$$

b)

$$F(s) = \frac{s^3 - 2s + 4}{s^4 - 2s^3}.$$

c)

$$F(s) = (s-3)^{-5}.$$