Exercise set 09

March 31, 2021

1 Homework assignment 9 (Lightweight Eastern edition)

This homework assignment considers only the disease spreading models and their numerical solutions already discussed in the interactive lecture from Wednesday

Deadline: Monday, 15th of April, 12:00.

[1]:
```
%matplotlib widget

import ipywidgets as widgets
from ipywidgets import interact, fixed
import numpy as np
from numpy import pi
from numpy.linalg import solve, norm
import matplotlib.pyplot as plt

# Use a funny plotting style
plt.xkcd()

newparams = {'figure.figsize': (6.0, 6.0),
             'axes.grid': True,
             'lines.markersize': 8,
             'lines.linewidth': 2,
             'font.size': 14}
plt.rcParams.update(newparams)
```

1.1 Problem 1 SIHR model

a) Write down the final ODE system for the SIHR model developed during the first breakout session. Denote the rates for the transition $I \xrightarrow{\gamma_h} H$, $I \xrightarrow{\gamma_r} R$, and $H \xrightarrow{\delta} R$ by γ_h and γ_r, δ, respectively.

b) As the SIHR model is a refinement of the SIR model where we want to distinguish between infectious and hospitialized individuals, we assume the same total γ as for the simpler SIR model; that is,

$$\gamma_r + \gamma_h =: \gamma = 1/18$$

Assume that * that 3.5% of all infected individuals will be hospitalized which means that $\gamma_h = 0.035\gamma$ * hospitalized individuals stay 14 days in the hospital on average, that is $\delta = 1/14$ * St. Olav's hospital has roughly 1000 beds with roughly 80% of them being occupied

Use the same initial condition as in the Trondheim scenario as in the lecture, choose a tolerance tol such that your numerical solutions are not off by more than 10 persons. Now compute and plot the solution S, I, H, R using both the adaptive Euler-Heun and Fehlberg's method. Record and state the number of steps you needed with Euler-Heun vs Fehlberg. What is (approximately) the largest basic reproduction number R_0 for which we will not exceed the maximal number of available beds?

Hint Before you start, it might be helpful to review the Some of the code snippets provided below might be helpful, as wee

[2]:
```
# define SIHR class similar to SIR before
class SIHR:
    # Create model with given parameters
    def __init__(self, beta, gamma_r, gamma_h, delta):
        self.beta = beta # infectional rate
        self.gamma_r = gamma_r # removal rate
        self.gamma_h = gamma_h # removal rate
        self.delta = delta

    def __call__(self, t, y):
        # TODO: Implement rhs of SIHR model
        return ...
```

[3]:
```
from rkm import EmbeddedExplicitRungeKutta

#Heun-Euler
# TODO: Fill in missing steps indicated by ...
a    = ...
bhat = ...
b    = ...
c = ...
order = ...

euler_heun = EmbeddedExplicitRungeKutta(a, b, c, bhat, order)

#Fehlberg method combines a 4th and 5th order method
a    = ...
bhat = ...
b    = ...
c = ...
order = 4
fehlberg = EmbeddedExplicitRungeKutta(a, b, c, bhat, order)

# max number of steps
Nmax = 10000
tol = ...
```

1.2 Problem 2 SIHRt model

Redo problem 1, but this time develop and use an extension the SIHR model to account for time-limited immunity, assuming 1 year of immunity for each recovered person. Consider a time-period of 5 years and find out how many "infection waves" will occur where the maximum capacity of beds are exceeded.