

Formula Sheet.

TMA4125/30/35 Matematikk 4N/D, Summer 2022.

Fourier Transform. The Fourier Transform $\hat{f} = \mathcal{F}(f)$ and its inverse $f = \mathcal{F}^{-1}(\hat{f})$ are

$$\hat{f}(\omega) = \mathcal{F}(f)(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)e^{-i\omega x} dx \quad \text{and} \quad f(x) = \mathcal{F}^{-1}(\hat{f})(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(\omega)e^{i\omega x} d\omega$$

Laplace Transform. The Laplace transform $F(s)$ of $f(t)$, $t \geq 0$, reads

$$F(s) = \int_0^{\infty} e^{-st} f(t) dt$$

List of Fourier Transforms.		List of Laplace Transforms.	
$f(x)$	$\hat{f}(\omega)$	$f(t)$	$F(s)$
e^{-ax^2}	$\frac{1}{\sqrt{2a}} e^{-\frac{\omega^2}{4a}}$	$\cos(\omega t)$	$\frac{s}{s^2 + \omega^2}$
$e^{-a x }$	$\sqrt{\frac{2}{\pi}} \frac{a}{\omega^2 + a^2}$	$\sin(\omega t)$	$\frac{\omega}{s^2 + \omega^2}$
$\frac{1}{x^2 + a^2}$ for $a > 0$	$\sqrt{\frac{\pi}{2}} \frac{e^{-a \omega }}{a}$	$\cosh(\omega t)$	$\frac{s}{s^2 - \omega^2}$
$\begin{cases} 1 & \text{for } x < a \\ 0 & \text{otherwise.} \end{cases}$	$\sqrt{\frac{2}{\pi}} \frac{\sin(\omega a)}{\omega}$	$\sinh(\omega t)$	$\frac{\omega}{s^2 - \omega^2}$
		t^n	$\frac{\Gamma(n+1)}{s^{n+1}}$, see Note ^(a)
		e^{at}	$\frac{1}{s-a}$
		$f(t-a)u(t-a)$	$e^{-sa}F(s)$
		$\delta(t-a)$	e^{-sa}

^(a) where for $n \in \mathbb{N}$ we have $\Gamma(n+1) = n!$

Trigonometric identities.

- $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$
- $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$
- $\sin \alpha \cos \beta = \frac{1}{2}(\sin(\alpha - \beta) + \sin(\alpha + \beta))$
- $\cos(2\alpha) = 2 \cos^2(\alpha) - 1 = 1 - 2 \sin^2(\alpha)$
- $2 \sin \alpha \cos \beta = \sin(\alpha + \beta) + \sin(\alpha - \beta)$
- $2 \cos \alpha \sin \beta = \sin(\alpha + \beta) - \sin(\alpha - \beta)$
- $2 \cos \alpha \cos \beta = \cos(\alpha - \beta) + \cos(\alpha + \beta)$
- $2 \sin \alpha \sin \beta = \cos(\alpha - \beta) - \cos(\alpha + \beta)$

We also discussed the sinus cardinalis $\text{sinc}(x) = \frac{\sin x}{x}$.

Fourier Series. For a 2π -periodic function f we can write

$$f \sim \sum_{k=-\infty}^{\infty} c_k e^{ikx} = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$$

with coefficients

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx, \quad n = 0, 1, 2, \dots, \quad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx, \quad n = 1, 2, \dots,$$

$$c_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-ikx} dx, \quad k \in \mathbb{Z}.$$

Order conditions for Runge-Kutta methods

p	Conditions
1	$\sum_{i=1}^s b_i = 1$
2	$\sum_{i=1}^s b_i c_i = \frac{1}{2}$
3	$\sum_{i=1}^s b_i c_i^2 = \frac{1}{3}$ $\sum_{i=1}^s \sum_{j=1}^s b_i a_{ij} c_j = \frac{1}{6}$
4	$\sum_{i=1}^s b_i c_i^3 = \frac{1}{4}$ $\sum_{i=1}^s \sum_{j=1}^s b_i c_i a_{ij} c_j = \frac{1}{8}$ $\sum_{i=1}^s \sum_{j=1}^s b_i a_{ij} c_j^2 = \frac{1}{12}$ $\sum_{i=1}^s \sum_{j=1}^s \sum_{k=1}^s b_i a_{ij} a_{jk} c_k = \frac{1}{24}$