LECTURE 3

Laplace Transform

continuation

- Heaviside function, δ-function
  - Recall definition of the Heaviside function and shifts
  - Formulation of the t-shift theorem (both direct and inverse transforms)
  - Proof
  - Examples
    - $\mathcal{L}(tu(t-2))$;
    - $$\mathcal{L}^{-1}\frac{se^{-2s}}{s^2+4s+8}$$
  - Recall: physical meaning of the right-hand side in the 2-nd order equation, forse, impulse
  - $\mathcal{L}\Big(h(u(t-a)-u(t-(a+\epsilon))\Big)$
  - Instant impulse, pictures, passing to the limit after Laplace transform
  - Definition: $\delta-function$, integration of δ-function against continuous function.
  - Example: Damped systems:
    - $y''+2y'+2y=\delta(t-1),\ y(0)=0,\ y'(0)=0$;
    - $y''+2y'+2y=\delta(t-1),\ y(0)=0,\ y'(0)=0$;
- Convolution
  - Definition of convolution
  - Two motivations:
    - inverse transform of the products;
    - systems with "weak memory"
  - Properties of convolution:
    - $f*(ag+bh)=a(f*g)+b(g*h)$

    - $(f * g)(t) = (g * f)(t)$
- Laplace transform of convolution
- back to differential equations considerd above,
- Integral equations:
    * Example: $y(t) = t + \int_0^t y(\tau) \sin(t - \tau) d\tau$