

LECTURE 4 Laplace Transform (*cont*).

Convolution

- Recall definition

$$\textit{Example: } f(t) = e^{at}, g(t) = e^{bt}$$

- $f * g = g * f$
- $\mathcal{L}(f * g) = F(s)G(s)$

- Application I: Integral equations:

$$\textit{Example: } y(t) = 1 + \int_0^t (t - \tau)y(\tau)d\tau$$

- Application II: initial value problem from equilibrium position

$$\textit{Example: } \text{Damped oscillations}$$

Differentiation and integration of the Laplace transform

- Differentiation of the LT
- Integration of LT
- Examples

$$\mathcal{L}^{-1}\left(\frac{s}{(\omega^2 + s^2)^2}\right) = \frac{t}{2\omega} \sin \omega t$$

$$\mathcal{L}^{-1} \log\left(1 + \frac{\omega^2}{s^2}\right) = \frac{2}{t}(1 - \cos \omega t)$$

Summary