LECTURE 8

FOURIER SERIES *(final lecture)*

Complex Fourier series

- Definition
- Relation to the trigonometric Fourier series
- Orthogonality relations for exponential functions
- Formulas for the coefficients
- Examples:
  - $f(t) = t^2$, $-\pi < t, \pi$,
  - $f(t) = (e^{it} + e^{3it})^2$; comments about similar situation for trigonometric Fourier series.
- Complex Fourier series of even and odd functions
- Decay of the coefficients

Approximation by trigonometric polynomials

- Recall geometric interpretation, basic relations
- Energy – analog of the length;
- Basic relations for Fourier series:
  - norm equality (Parseval)
  - norm inequality (Bessel)
  - estimate of the remainder term
  - proof of the Parseval relation
  - reformulation as approximation problem: minimal square error
  - example: $f(t) = t$, $-\pi < t < \pi \ \Rightarrow$

$$\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$$

Forced oscillations

- setting of the problem
- idea of the solution
- example: take the right-hand side from the previous example
- approximate solution
- *Remark:* resonance situation