LECTURE 9

FOURIER INTEGRALS

Cosine and sine integrals

- Introduction, the main idea.
- Recall formulas for Fourier series with period $2L$
- Functions
$$A(w) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(t)\cos wt\ dt, \quad B(w) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(t)\sin wt\ dt$$
- Passing to the limit, inverse formula
$$f(t) = \int_0^{\infty} \Big(A(w)\cos wt + B(w)\sin wt\Big)\ dw$$
- Examples:
  - Square step
  - $f(t) = 0$, if $t < 0$; $f(t) = e^{-at}$, if $t > 0$.
- Convergence of the integrals
- Half-range expansions: $f(t)$, $t > 0$, $\Rightarrow$
$$A(w) = \frac{2}{\pi}\int_0^{\infty} f(t)\cos wt\ dt, \quad f(t) = \int_0^{\infty} A(w)\cos wt\ dw;$$
$$B(w) = \frac{2}{\pi}\int_0^{\infty} f(t)\sin wt\ dt, \quad f(t) = \int_0^{\infty} B(w)\sin wt\ dw.$$

Complex Fourier transform

- Basic formulas:
$$\mathcal{F}f(w) = \hat{f}(w) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)e^{-itw}\ dt, \quad f(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \hat{f}(w)e^{itw}\ dw,$$
- Analogy with complex Fourier series
- When all this does work
- Example:
$$\mathcal{F}\Big(e^{-ax^2})(w) = \frac{1}{\sqrt{2a}}e^{-w^2/4a}$$
  - differential equation and solution
  - finding the constant