LECTURE 10

FOURIER INTEGRALS

Complex Fourier transform

- Properties of Fourier transform
  - linearity
  - Fourier transform of even and odd functions
  - decay and derivatives
  - convergence of the Fourier transform
  - Parseval equality
- Convolution
  - definition of convolution
  - "physical" example
  - relation to convolution we have already dealt with
  - convolution and Fourier transform
- Examples *(old exam problems)*
  - Solve linear integral equation

$$\int_{-\infty}^{\infty} f(p)e^{-b(x-p)^2}\,dp = e^{-x^2}, \quad b > 1.$$

  - The functions $f(x)$ and $g(x)$ are given by the relations

$$f(x) = \begin{cases} 1, & \text{for } |x| < 1; \\ 0, & \text{otherwise.} \end{cases} \quad \text{and} \quad g(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & \text{otherwise.} \end{cases}$$

i Find Fourier transforms of f and g
ii Let $h = f * g$. Prove that

$$h(x) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{(1-iw)\sin w}{w(1+w^2)}\,dw$$

and find the value of the integral

$$\int_{-\infty}^{\infty} \frac{\sin w}{w(1+w^2)}\,dw$$

(You may assume that h is a continuous function).