

LECTURE 11

PARTIAL DIFFERENTIAL EQUATIONS

Introduction

- What is a partial differential equation. Linear equations with constant coefficients.
- Examples

- Wave equation:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

- Heat equation:

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

- Laplace and Poisson equations

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y)$$

- 2D Wave equation:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

- 3D Laplace equations

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$

- Initial and boundary conditions: cover all examples
- Linearity of equations $\Rightarrow$ (one of) basic idea: look for a solution as a sum of simpler ones (Separation of variables)

Separation of variables, wave equation

- Repeat setting of the problem; boundary conditions, initial conditions
- *Step 1* Product solutions; two ordinary differential equations
- *Step 2* Boundary conditions $\Rightarrow$ admissible values of constant. Elementary solutions

- *Step 3* Final solution: meeting initial conditions, final solution.
- Another boundary conditions, f.ex.

$$u_x(0, t) = 0, u_x(0, \pi) = 0$$