

## LECTURE 12

### PARTIAL DIFFERENTIAL EQUATIONS *cont.*

*Example from old exam* (continuation of the previous lecture):

$$\begin{aligned}u_{xx} &= u_{tt} - 4u, & 0 \leq x \leq \pi, t > 0; \\u(0, t) &= 0, u(\pi, t) = 0 \\u(x, 0) &= 0, \quad u_t(x, 0) = \sin x + \sin 2x + \sin 3x\end{aligned}$$

### Heat equation

- Deriving 1D heat equation.
- Heat equation on a segment
  - Meaning of the boundary values
  - Separation of variables, step 1: two ordinary differential equations
  - Separation of variables, step 2: meeting boundary conditions  $\Rightarrow$  admissible values of the spectral parameter
  - Separation of variables: final solution
  - Discussion of the behavior of the solution
  - Insulated boundary conditions
  - *Example:* standard equation and  $u(x, 0) = 1 - x/\pi$  for  $0 < x < \pi$ .
  - Non-homogeneous boundary conditions.