



Contact during the exam:
Yura Lyubarskii (735 935 26)

EXAM IN CALCULUS 4N (TMA4125)

Monday June 06, 2005

Time: 09:00 -13:00

Hjelpemidler: Simple calculator (HP 30S), Rottmann: matematisk formelsamling

Grades: 27.06.05

Oppgave 1

- a) Find the inverse Laplace transform of the function

$$F(s) = \frac{s+4}{(s+2)^2}$$

- b) Solve the initial value problem:

$$y''(t) + 4y'(t) + 4y(t) = 0, \quad t \geq 0, \quad y(0) = 1, \quad y'(0) = 0.$$

- c) Solve the integral equation:

$$y(t) + \int_0^t e^{-2(t-\tau)} y(\tau) d\tau = e^{-2t}, \quad t > 0.$$

Oppgave 2

- a) Find Fourier series for 2π -periodic even function $f(t)$ such that $f(x) = \frac{\pi}{2} - x$, $0 < x < \pi$.

b) Find all solutions of the form $u(x, t) = X(x)T(t)$ for the problem:

$$(1) \frac{\partial^2 u}{\partial x^2} - 2u - \frac{\partial u}{\partial t} = 0, \quad 0 < x < \pi, \quad t > 0.$$

$$(2) u_x(0, t) = 0, \quad u_x(\pi, t) = 0, \quad t > 0.$$

c) Find the solution to the problem (1), (2) in part b.) which also satisfies the initial condition

$$u(x, 0) = \frac{\pi}{2} - x, \quad 0 < x < \pi.$$

Oppgave 3

Find complex Fourier transform of the function $f(x) = e^{-|x|}$ and then find the value of the integral

$$\int_{-\infty}^{\infty} \frac{\cos \omega}{1 + \omega^2} d\omega$$

Oppgave 4

a) Find a polynomial of the smallest possible degree which solves the interpolation problem

x_k	-2	-1	0	1	2
$p(x_k)$	6	0	0	0	15

b) Let $p(x)$ be the polynomial from part a). Using the Simpson method with step 1 evaluate the integral $\int_{-2}^2 p(x) dx$.

Oppgave 5

We are solving partial differential equation

$$u_t = u_{xx}, \quad -1 \leq x \leq 1, \quad t \geq 0$$

$$u(x, 0) = 1 - x^2,$$

$$u(-1, t) = 0, \quad u(1, t) = 0, \quad t \geq 0.$$

Let $k = 0.5, h = 0.5$. Using the Crank-Nikolson method write down the system of linear equations for the values

$$u_{11} \approx u(-0.5, 0.5), \quad u_{21} \approx u(0, 0.5), \quad u_{31} \approx u(0.5, 0.5).$$