Exercise #10

26. October 2022

Problem 1.

Compute the Laplace transform of the following functions.

a) $f(t) = 5 + t^3 - 4t^6$.

b) $f(t) = te^{2t}$.

c) $f(t) = e^{-t}\cos(5t)$.

Problem 2.

Find the inverse Laplace transform of the following functions.

a) $F(s) = -\frac{4}{s^2} + \frac{3}{s^5}$.

b) $F(s) = \frac{s+13}{s^2 - 6s + 6}$.

c) $F(s) = \frac{2}{(s-1)(s^2+1)}$.

Problem 3.

Decide for each of the following statements whether it is true or false. Explain your answer.

a) If f and g are two functions for which the Laplace transform exists, then $\mathcal{L}(f-g) = \mathcal{L}(f) - \mathcal{L}(g)$.

b) If f and g are two functions, for which the Laplace transform exists, then $\mathcal{L}(f \cdot g) = \mathcal{L}(f) \cdot \mathcal{L}(g)$.

c) If the function f satisfies $0 \leq f(t)$ for all $t \geq 0$, then $\mathcal{L}(f)(s) \geq 0$ for all s for which $\mathcal{L}(f)(s)$ exists.

d) If the function f is continuous and satisfies $0 \leq f(t) \leq 1$ for all $t \geq 0$, then $\mathcal{L}(f)(s)$ exists for all $s > 0$.

Problem 4.

Use the Laplace transform in order to solve the following initial value problems:

a) $y'' + y' - 6y = 0,\ y(0) = 1,\ y'(0) = 1.$

b) $y''' + y' = 1,\ y(0) = 1,\ y'(0) = -1,\ y''(0) = -1.$

c) $y'' + 5y' + 6y = 0,\ y(0) = -2,\ y'(0) = 1.$

The next exercises are optional and should not be handed in!

Problem 5.

Compute the Laplace transform of the following functions:

a) $f(t) = \sin(4t)e^{3t}.$

b) $f(t) = \sinh(t)\cos(t).$

Problem 6.

Compute the inverse Laplace transform of $F(s)$, where

a) $F(s) = \dfrac{1}{s^2 + s + 1}$,

b) $F(s) = \dfrac{as}{s^2 - 2as + a^2 + 1}$ for some $a \in \mathbb{R}$,

c) $F(s) = \dfrac{1}{s^3 + s}$.

Problem 7.

a) Use the Laplace transform to solve the differential equation,

$$y' = -2y + 1, \quad y(0) = 0.$$

b) Use the Laplace transform to solve the differential equation,

$$y''' = -y' + 2, \quad y(0) = y'(0) = y''(0) = 0.$$