

## Formler i numerikk

- La  $p(x)$  være et polynom av grad  $\leq n$  som interpolerer  $f(x)$  i punktene  $x_i, i = 0, 1, \dots, n$ . Forutsatt at  $x$  og alle nodene ligger i intervallet  $[a, b]$ , så gjelder

$$f(x) - p(x) = \frac{1}{(n+1)!} f^{(n+1)}(\xi) \prod_{i=0}^n (x - x_i)$$

Hvis nodene er jevnt fordelt (inkludert endepunktene), og  $|f^{(n+1)}(x)| \leq M$ , da gjelder

$$|f(x) - p(x)| \leq \frac{1}{4(n+1)} M \left(\frac{b-a}{n}\right)^{n+1}$$

- Numerisk derivasjon:

$$f'(x) = \frac{1}{h}(f(x+h) - f(x)) - \frac{1}{2}hf''(\xi)$$

$$f'(x) = \frac{1}{2h}(f(x+h) - f(x-h)) - \frac{h^2}{6}f''(\xi)$$

$$f''(x) = \frac{1}{h^2}(f(x+h) - 2f(x) + f(x-h)) - \frac{h^2}{12}f^{(4)}(\xi)$$

- Newtons metode for ligningssystemet  $\mathbf{f}(\mathbf{x}) = \mathbf{0}$  er gitt ved

$$\begin{aligned} \mathbf{J}^{(k)} \cdot \Delta \mathbf{x}^{(k)} &= -\mathbf{f}(\mathbf{x}^{(k)}) \\ \mathbf{x}^{(k+1)} &= \mathbf{x}^{(k)} + \Delta \mathbf{x}^{(k)} \end{aligned}$$

- Iterative teknikker for løsning av et lineært ligningssystem

$$\sum_{j=1}^n a_{ij}x_j = b_i, \quad i = 1, 2, \dots, n$$

$$\text{Jacobi: } x_i^{(k+1)} = \frac{1}{a_{ii}} \left( b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k)} - \sum_{j=i+1}^n a_{ij}x_j^{(k)} \right)$$

$$\text{Gauss-Seidel: } x_i^{(k+1)} = \frac{1}{a_{ii}} \left( b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k+1)} - \sum_{j=i+1}^n a_{ij}x_j^{(k)} \right)$$

- Heuns metode for løsning av  $\mathbf{y}' = \mathbf{f}(x, \mathbf{y})$ :

$$\mathbf{K}_1 = h\mathbf{f}(x_n, \mathbf{y}_n)$$

$$\mathbf{K}_2 = h\mathbf{f}(x_n + h, \mathbf{y}_n + \mathbf{K}_1)$$

$$\mathbf{y}_{n+1} = \mathbf{y}_n + \frac{1}{2}(\mathbf{K}_1 + \mathbf{K}_2)$$

Se også formlene i Rottmann.

**Tabell over Laplacetransformerte**

$f(t)$	$\mathcal{L}(f)$
1	$\frac{1}{s}$
$t$	$\frac{1}{s^2}$
$t^n$ ( $n = 0, 1, 2, \dots$ )	$\frac{n!}{s^{n+1}}$
$e^{at}$	$\frac{1}{s-a}$
$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
$\cosh at$	$\frac{s}{s^2 - a^2}$
$\sinh at$	$\frac{a}{s^2 - a^2}$
$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2 + \omega^2}$
$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2 + \omega^2}$