**Norges teknisk-naturvitenskapelige universitet**
**Institutt for matematiske fag**



TMA4125 Calculus 4N
English
March 30, 17:15-18:15, Room R 2
Midterm Exam

**Problem 1.** Let $z_1 = 1 + i$, $z_2 = 1 + 2i$. The real part of $z_1 \overline{z_2}$ equals

a $1$, b $2$, c $3$, d $4$.

**Problem 2.** The Laplace transform of $u(t-2)\sin(t-1)$ (here $u$ is the Heaviside function) equals

a $\frac{1}{s^2+1}(\cos 1 + s\sin 1)$, b $\frac{e^{-2s}}{s^2+1}(s\cos 1 + \sin 1)$,
c $\frac{e^{-2s}}{s^2+1}(\cos 1 + s\sin 1)$, d $\frac{e^{-s}}{s^2+4}(s\cos 2 + \sin 2)$.

**Problem 3.** The inverse Laplace transform of $\frac{1}{(s+2)^2}$ is

a $te^{-2t}$, b $t^2e^{-2t}$, c $t^2e^{-t}$, d $t^2$.

**Problem 4.** Solution to the equation

$$y(t) = 2 - \int_0^t y(\tau)d\tau, \quad t > 0$$

is

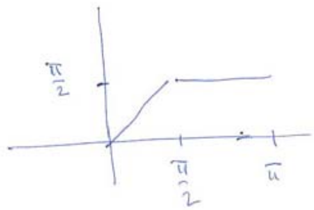
a $e^{-2t}$, b $2e^{-t}$, c $2e^{t}$, d $(t+1)e^{-3t}$.

**Problem 5.** Consider the Fourier series expansion of $2\pi$ periodic function $f(x) = (\cos x + 1)^2 \sin^3 x$. The $a_0$ coefficient of this expansion equals

a $2$, b $1$, c $0$, d $-1$.



**Problem 6.** Let on the segment $[0, \pi]$ the function $f(x)$ be represented by the graph below,




and let $f(x) \asymp \sum_1^\infty b_n \sin nx$ be its sine half-range expansion. The sum of this series at the point $x = 3\pi$ is

a $\frac{\pi}{2}$, b $1$, c $-1$, d $0$.

**Problem 7.** Which of the functions below is a solution to the following problem:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \quad t > 0$$

$$u(0,t) = u(\pi, t) = 0.$$

a $e^{-t}\cos x$, b $e^{-2t}\sin 2x$, c $e^{-9t}\cos 2x$, d $e^{-9t}\sin 3x$.

**Problem 8.** Let

$$f(x) = \begin{cases} 1, & 2 < x < 4; \\ 0, & \text{otherwise.} \end{cases}$$

The Fourier transform of $f$ is

a $\sqrt{\frac{2}{\pi}}\frac{\sin w}{w}e^{-3iw}$, b $\sqrt{\frac{2}{\pi}}\frac{\cos w}{w}e^{-3iw}$,
c $\sqrt{\frac{\pi}{2}}(e^{-3iw} - e^{-iw})$, d $\frac{1}{\pi}\frac{\cos w - 1}{w}$.

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TMA4125 Calculus 4N
English
March 9, 15:15-16:15, Room R 2
Midterm Exam

**Problem 1**

Let $z_1 = 1 + i, z_2 = 1 - i$. Find $Im\frac{z_1}{z_2}$.

Answers:

(a) 1 (b) $\sqrt{2}$ (c) $i$ (d) $-1$

**Problem 2**

Find the inverse Laplace transform of the function $\frac{-s}{s^2+6s+8}$.

(a) $2e^{-2t} - e^{-4t}$ (b) $e^{-4t} - 4e^{2t}$ (c) $e^{-2t} - 2e^{-4t}$ (d) $e^{2t} - 2e^{4t}$

**Problem 3**

Given the initial value problem

$y''(t) - y(t) = 0,\ t > 0;\ y(0) = 1,\ y'(0) = 2.$

The Laplace transform of the solution $y(t)$ is

(a) $\frac{2-s}{s^2+1}$ (b) $\frac{s^2}{s+1}$ (c) $\frac{s}{s+2}$ (d) $\frac{2+s}{s^2-1}$.



**Problem 4**

The Laplace transform of the function $e^t u(t-1)$ is:

(a) $\frac{e^s}{s+1}$ (b) $\frac{e^{-(s-1)}}{s-1}$ (c) $\frac{e^{s-1}}{s-2}$ (d) $\frac{e^{-(s-1)}}{s^2}$

**Problem 5**

An odd $2\pi$−periodic function $f$ is defined by

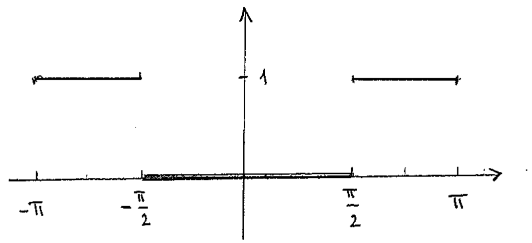
$f(x) = \cos^2 x,\ 0 < x < \pi$.

Let $f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\,x + b_n \sin\, x)$ be its Fourier series. The value of the coefficient $a_0$ is:

(a) $\frac{1}{2}$ (b) 0 (c) $-1$ (d) $-\frac{1}{2}$

**Problem 6**

Let $f(x)$ be a $2\pi$−periodic function whose value on $(-\pi, \pi)$ is defined from the graph




The Fourier series of $f$ at the point $x = \frac{3\pi}{2}$ converges to

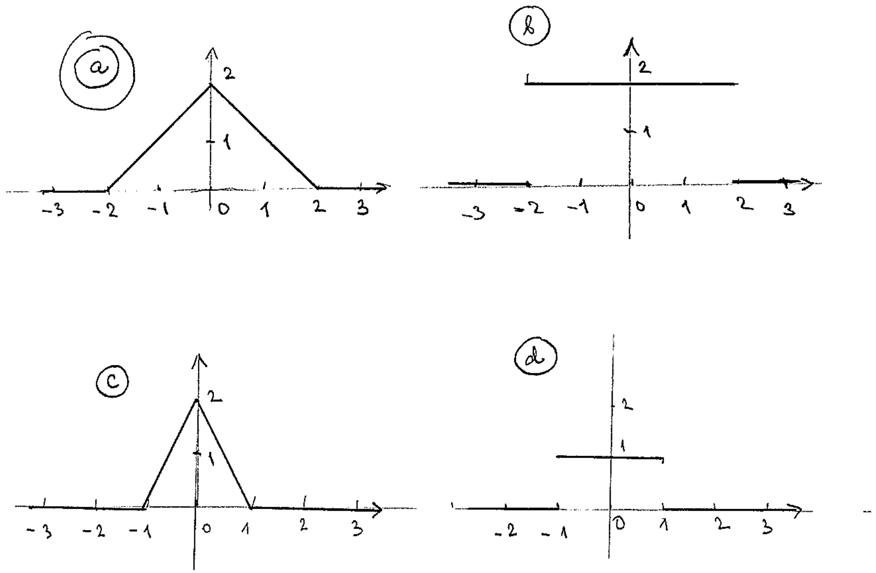
(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\pi$

**Problem 7**

Let $f(t)$ be given by the formula

$$f(t) = \begin{cases} 1 & -1<t<1; \\ 0 & \text{otherwise.} \end{cases}$$

Which of the pictures below is the graph of $g(t) = (f * f)(t)$?




**Problem 8**

Which of the functions below is a solution of the partial differential equation $u_x - u_y = 0$?

(a) $\frac{1}{2}\cos\, x - \frac{1}{2}\sin\, y$ (b) $\frac{1}{2}\cos\, x\, \cos\, y - \frac{1}{2}\sin\, x \sin\, y$

(c) $x^2 - 2xy + y^2$ (d) $(x+y)e^{x-y}$