TMA4125 Calculus 4N
English
March 30, 17:15-18:15, Room R 2
Midterm Exam

Problem 1. Let $z_1 = 1 + i$, $z_2 = 1 + 2i$. The real part of $z_1 \overline{z_2}$ equals

a 1, b 2, (c) 3, d 4.

Problem 2. The Laplace transform of $u(t - 2)\sin(t - 1)$ (here u is the Heaviside function) equals

a $\dfrac{1}{s^2 + 1}(\cos 1 + s \sin 1)$, b $\dfrac{e^{-2s}}{s^2 + 1}(s \cos 1 + \sin 1)$,

(c) $\dfrac{e^{-2s}}{s^2 + 1}(\cos 1 + s \sin 1)$, d $\dfrac{e^{-s}}{s^2 + 4}(s \cos 2 + \sin 2)$.

Problem 3. The inverse Laplace transform of $\frac{1}{(s+2)^2}$ is

(a) te^{-2t}, b $t^2 e^{-2t}$, c $t^2 e^{-t}$, d t^2.

Problem 4. Solution to the equation

$$y(t) = 2 - \int_0^t y(\tau)d\tau, \quad t > 0$$
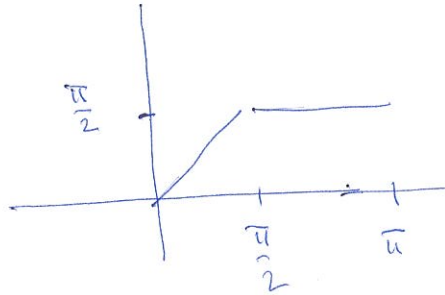
is

a e^{-2t}, (b) $2e^{-t}$, c $2e^t$, d $(t+1)e^{-3t}$.

Problem 5. Consider the Fourier series expansion of 2π periodic function $f(x) = (\cos x + 1)^2 \sin^3 x$. The a_0 coefficient of this expansion equals

a 2, b 1, (c) 0, d -1.

Problem 6. Let on the segment $[0, \pi]$ the function $f(x)$ be represented by the graph below,

and let $f(x) \asymp \sum_1^\infty b_n \sin nx$ be its sine half-range expansion. The sum of this series at the point $x = 3\pi$ is

$$\textbf{a} \quad \frac{\pi}{2}, \qquad \textbf{b} \quad 1, \qquad \textbf{c} \quad -1, \qquad \boxed{\textbf{d}} \quad 0.$$

Problem 7. Which of the functions below is a solution to the following problem:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < \pi, \quad t > 0$$

$$u(0, t) = u(\pi, t) = 0.$$

$$\textbf{a} \quad e^{-t}\cos x, \qquad \textbf{b} \quad e^{-2t}\sin 2x, \qquad \textbf{c} \quad e^{-9t}\cos 2x, \qquad \boxed{\textbf{d}} \quad e^{-9t}\sin 3x.$$

Problem 8. Let

$$f(x) = \begin{cases} 1, & 2 < x < 4; \\ 0, & \text{otherwise.} \end{cases}$$

The Fourier transform of f is

$$\boxed{\textbf{a}} \quad \sqrt{\frac{2}{\pi}}\frac{\sin w}{w}e^{-3iw}, \qquad \textbf{b} \quad \sqrt{\frac{2}{\pi}}\frac{\cos w}{w}e^{-3iw},$$

$$\textbf{c} \quad \sqrt{\frac{\pi}{2}}(e^{-3iw} - e^{-iw}), \qquad \textbf{d} \quad \frac{1}{\pi}\frac{\cos w - 1}{w}.$$

TMA4125 Calculus 4N
English
March 9, 15:15-16:15, Room R 2
Midterm Exam

Problem 1

Let $z_1 = 1 + i, z_2 = 1 - i$. Find $Im\frac{z_1}{z_2}$.

Answers:

(a) 1   (b) $\sqrt{2}$ (c) i (d) -1

Problem 2

Find the inverse Laplace transform of the function $\dfrac{-s}{s^2 + 6s + 8}$.

(a) $2e^{-2t} - e^{-4t}$ (b) $e^{-4t} - 4e^{2t}$ (c) $e^{-2t} - 2e^{-4t}$ (d) $e^{2t} - 2e^{4t}$

Problem 3

Given the initial value problem

$y''(t) - y(t) = 0, \ t > 0; \ y(0) = 1, \ y'(0) = 2.$

The Laplace transform of the solution $y(t)$ is

(a) $\dfrac{2-s}{s^2+1}$ (b) $\dfrac{s^2}{s+1}$ (c) $\dfrac{s}{s+2}$ (d) $\dfrac{2+s}{s^2-1}$.

Problem 4

The Laplace transform of the function $e^t u(t-1)$ is:

(a) $\dfrac{e^s}{s+1}$ (b) $\dfrac{e^{-(s-1)}}{s-1}$ (c) $\dfrac{e^{s-1}}{s-2}$ (d) $\dfrac{e^{-(s-1)}}{s^2}$

Problem 5

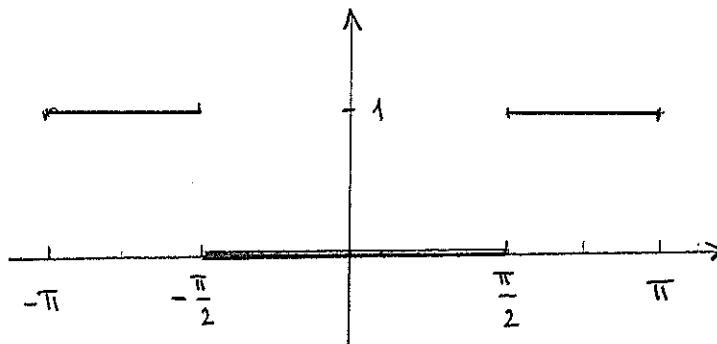An odd 2π–periodic function f is defined by

$f(x) = \cos^2 x,\ 0 < x < \pi.$

Let $f(x) = a_0 + \sum\limits_{n=1}^{\infty}(a_n \cos n\,x + b_n \sin x)$ be its Fourier series. The value of the coefficient a_0 is:

(a) $\frac{1}{2}$ (b) 0 (c) -1 (d) $-\frac{1}{2}$

Problem 6

Let $f(x)$ be a 2π–periodic function whose value on $(-\pi, \pi)$ is defined from the graph

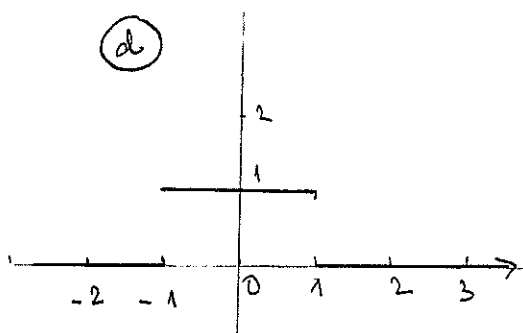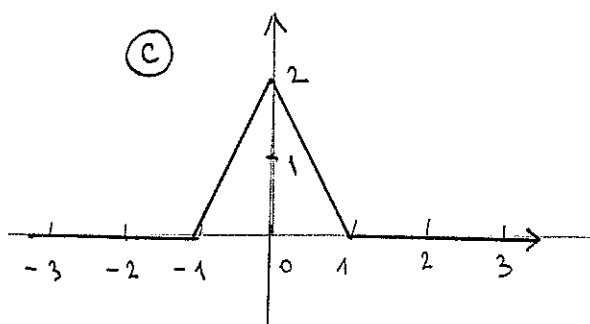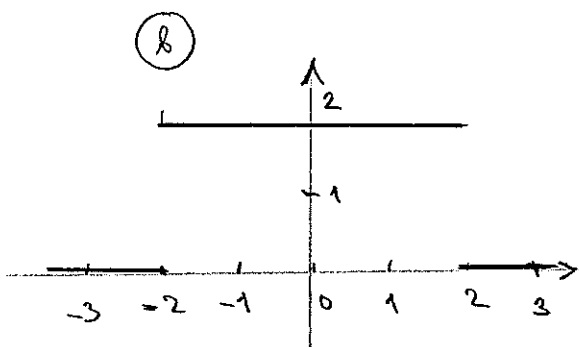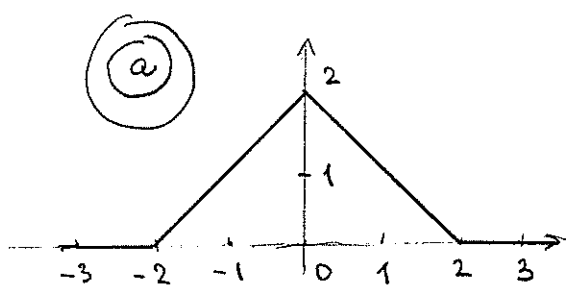The Fourier series of f at the point $x = \frac{3\pi}{2}$ converges to

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) π

Problem 7

Let $f(t)$ be given by the formula

$$f(t) = \begin{cases} 1 & -1<t<1; \\ 0 & \text{otherwise.} \end{cases}$$

Which of the pictures below is the graph of $g(t) = (f * f)(t)$?

Problem 8

Which of the functions below is a solution of the partial differential equation $u_x - u_y = 0$?

(a) $\frac{1}{2}\cos x - \frac{1}{2}\sin y$ (b) $\frac{1}{2}\cos x \cos y - \frac{1}{2}\sin x \sin y$

(c) $x^2 - 2xy + y^2$ (d) $(x+y)e^{x-y}$