NTNU
Institutt for matematiske fag

TMA4130 MATEMATIKK 4N
Midterm test Thursday October 27th 2005
Hours: 5:15pm – 6:45pm (90min)

Material allowed during the midterm test: Simple calculator (HP30S)
Rottman: *Booklet of formula*

N.B: Use only *one* cross for each of the exercises on the answer sheet. *Do not* write on the exercise sheet!

Exercise 1 A periodic function *f* with period 2 is defined as $f(x) = x^2$ for $-1 < x \leq 1$.

At the point x = 99,8 the Fourier series of *f* converge towards the value:

A: 0,16 **B:** −0,4 **C:** 9960,04 **D:** 0,04

Exercise 2 The Fourier coefficient *a1* for the function defined in exercise 1 is:

A: $\frac{1}{4\pi^2}$ **B:** $-\frac{4}{\pi^2}$ **C:** 0 **D:** $\frac{2}{\pi}$

Exercise 3 A function *f* with the period 2 is given by $f(x) = x^9$ for $-1 < x \leq 1$. At the point x = -9, the Fourier series of *f* converge towards the value:

A: 1 **B:** 0 **C:** $\frac{1}{2}$ **D:** -1

Exercise 4 The Laplacian transform of the function $t^2u(t-1)$ is:

A: $\frac{e^{-s}}{s^3}$ **B:** $e^{-s}\frac{s^2+2s+2}{s^3}$ **C:** $e^{-s}\frac{s-1}{s^3}$ **D:** $e^{1-s}\frac{2}{s^3}$

Exercise 5 The inverse Laplacian transform of $\frac{e^{-\pi s}}{s^2+2s+5}$ is

A: $u(t-\pi)\sin t$ **B:** $e^{\pi-t}\cos t$

C: $(t-\pi)u(t-\pi)\sin 2t$ **D:** $u(t-\pi)e^{\pi-t}\frac{1}{2}\sin 2t$

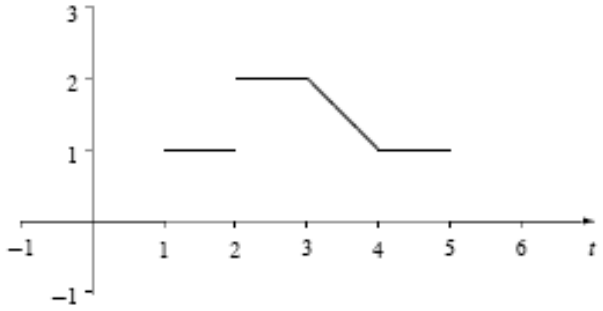
Exercise 6 The solution *y(t)* of the initial value problem

$$y'' + y = 3\cos 2t, \qquad y(0) = y'(0) = 0,$$

has the Laplacian transformed *Y(s)* given by:

A: $\dfrac{3s}{(s^4 + 5s^2 + 4)}$ **B:** $\dfrac{6}{(s^2+1)(s^2+4)}$ **C:** $\dfrac{3e^{-2s}}{s^2+1}$ **D:** $\dfrac{3}{s(s^2+1)}$

Exercise 7 The function with the given graph



is given by:

A: $u(t-1) + u(t-2) - (t-3)u(t-3) - u(t-5)$

B: $u(t-1) + u(t-2) - (t-3)u(t-3) + (t-4)u(t-4) - u(t-5)$

C: $u(t-1) + u(t-2) - (t-3)u(t-3) + tu(t-4) - u(t-5)$

D: $u(t-1) + u(t-2) - tu(t-3) + tu(t-4) - u(t-5)$

Exercise 8

The convolution product (refer to page 176 in Rottmann, the booklet of formulas,, for definition of the convolution product) 1 * *cos t* is equal to

A: $t\cos t$ **B:** $\cos t$ **C:** $\sin t$ **D:** te^{-t}

Exercise 9

Let the function *f(x)* be given by

$$f(x) = \begin{cases} 1 & \text{for } |x| \le 1 \\ 0 & \text{ellers.} \end{cases}$$

The Fourier transformed $\hat{f}(w)$ is given by:

A: $\frac{\pi w}{1+w^2}$ **B:** $\frac{1}{\sqrt{2\pi}}\frac{e^{iw}-1}{w}$ **C:** $\sqrt{\frac{2}{\pi}}\frac{\sin w}{w}$ **D:** $\sqrt{\frac{2}{\pi}}\frac{\cos w}{w}$

Exercise 10

The value of the integration

$$\int_0^\infty \arctan\frac{2}{w^2}\,dw$$

is (*Hint:* Use that the Fourier transformation of $f(x) = \frac{e^{-|x|}\sin x}{x}$ is $\hat{f}(w) = \frac{1}{\sqrt{2\pi}}\arctan\frac{2}{w^2}$.
)

A: $\frac{\pi}{2}$ **B:** 2π **C:** π **D:** $\frac{\pi^3}{32}$