MIDTERM TEST – TMA4125 MATEMATIKK 4N

Thursday 16th of March 2006
Time: 10:15 am – 11:15 pm

Exercise 1

Which of the following functions is the Laplacian transformed of $e^{-t} * cost$?

A : $\frac{s}{(s^2+1)(s+1)}$ **B** : $\frac{1}{(s^2+1)(s+1)}$

C : $\frac{1}{(s^2-1)(s+1)}$ **D** : $\frac{s}{(s^2+1)(s-1)}$

Exercise 2

Which of the functions beneath is the Laplacian transformed to the solution of the following initial value problem:

$$y''(t) + 4y(t) = \delta(t-2),\ y(0) = 0,\ y'(0) = 0\,,$$

where δ is the Diracs δ-function.

A : $\frac{e^{2s}}{s^2+1}$ **B** : $\frac{e^{-s}}{s^2+4}$

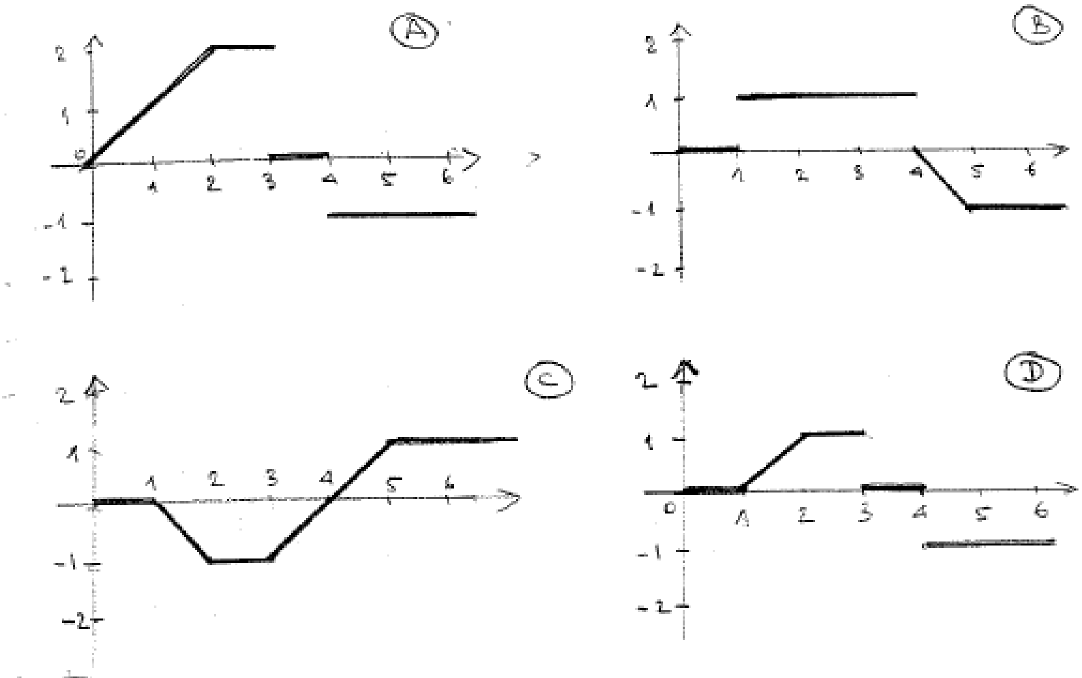
C : $\frac{e^{-2s}}{s^2+4}$ **D** : $\frac{e^{-2s}}{(s+2)^2}$

Exercise 3

Let

$$f(t) = (t-1)u(t-1) - (t-2)u(t-2) - u(t-3) - u(t-4)$$

where u is the Heaviside-function. Which of the pictures beneath is the graph of f?



Exercise 4

Look at the 2π-periodic function $f(t) = cos^2 t$. Let

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{int}$$

be its expansion in a complex Fourier series. The coefficient c_2 will then be equal to

A : 1 **B** : $\frac{1}{2}$

C : $-\frac{1}{2}$ **D** : $\frac{1}{4}$

Exercise 5

Let *f* be an even 2π-periodic function such that $f(t) = \frac{t^2}{2}$ for $0 \leq t \leq \pi$. The value *f*(7) is then

$\mathbf{A}:\quad 1$ $\qquad\qquad$ $\mathbf{B}:\quad 0$

$\mathbf{C}:\quad \frac{1}{2}(3\pi - 7)^2$ $\qquad\qquad$ $\mathbf{D}:\quad \frac{1}{2}(7 - 2\pi)^2$

Exercise 6

Let *f* be the function defined in the previous exercise (exercise 5). It is known that its Fourier series has the form

$$f(t) = \frac{\pi^2}{6} + 2\sum_{1}^{\infty} \frac{(-1)^n}{n^2} \cos nt.$$

The sum $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{4} + \ldots,$ (which means $\sum_1^\infty \frac{1}{n^2}$) is equal to

$\mathbf{A}:\quad \frac{\pi}{3}$ $\qquad\qquad$ $\mathbf{B}:\quad \frac{\pi^2}{3}$

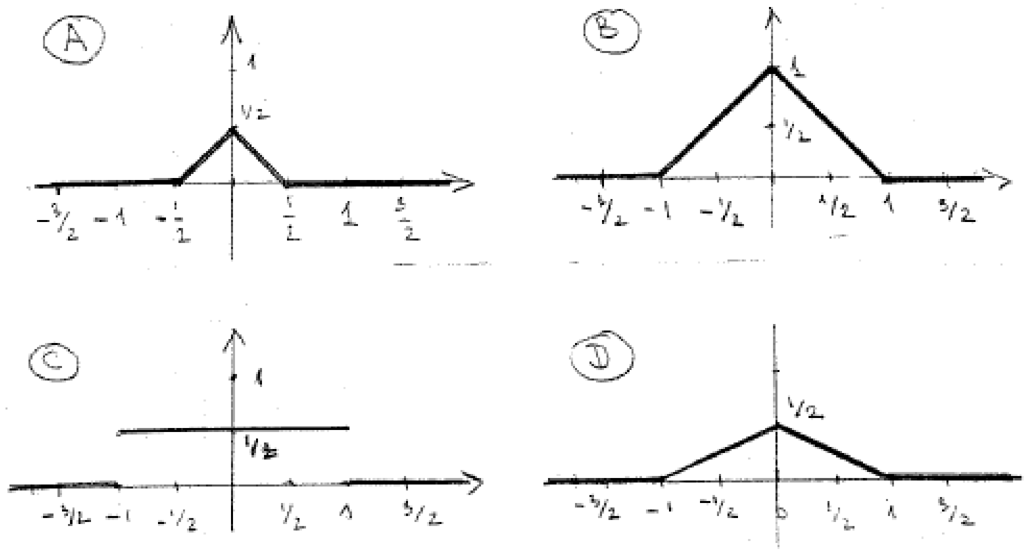
$\mathbf{C}:\quad \frac{\pi^2}{4}$ $\qquad\qquad$ $\mathbf{D}:\quad \frac{\pi^2}{6}$

Exercise 7

Let

$$f(t) = \begin{cases} 1, & \text{hvis } |t| \leq 1/2; \\ 0, & \text{ellers.} \end{cases}$$

Which of the pictures beneath is the graph of $f * f$?



Exercise 8

Let

$$f(t) = \begin{cases} 1 - |t|, & \text{hvis } |t| \leq 1; \\ 0, & \text{ellers.} \end{cases}$$

The Fourier transformed (in complex form) of *f* is

$\mathbf{A}:\quad \dfrac{2}{\pi}\left(\dfrac{\sin(w/2)}{w}\right)^2$

$\mathbf{B}:\quad \dfrac{\cos w - 1}{w^2}$

$\mathbf{C}:\quad \pi e^{-w^2/2}$

$\mathbf{D}:\quad \dfrac{\sin(w/2)}{w}$