

Øving 1

I.4.5

Gruppa oppfyller ikke Gj1:

$$(a * b) * c = \frac{a}{b} * c = \frac{\frac{a}{b}}{c} = \frac{a}{bc}$$

$$a * (b * c) = a * \frac{b}{c} = \frac{a}{\frac{b}{c}} = \frac{ac}{b}$$

Ser at $(a * b) * c \neq a * (b * c)$

I.4.8

*	1	3	5	7
1	1	3	5	7
3	3	1	7	5
5	5	7	1	3
7	7	5	3	1

Observer at tabellen er symmetrisk om diagonalen.

I.4.18

$$G = \{ A \text{ } n \times n \text{ matrise} \mid \det A = \pm 1 \}$$

$$\text{La } A, B \in G$$

$$\text{Da er } A * B = AB$$

$$\det AB = \det A \cdot \det B = (\pm 1) \cdot (\pm 1) = \pm 1$$

$\Rightarrow AB \in G \Rightarrow A * B \in G$, så $*$ er en binær operasjon.

$$\begin{aligned} \text{Gj1: } (A * B) * C &= AB * C = (AB)C = A(BC) \\ &= A * (BC) = A * (B * C) \end{aligned}$$

Gj2: I er identitets elementet i gruppa.

$$A * I = AI = A = IA = I * A$$

Gj3: Siden $\det A \neq 0 \quad \forall A \in G \Rightarrow \exists A^{-1}$
slik at $AA^{-1} = A^{-1}A = I$.

$\Rightarrow G$ er en gruppe.

I.4.32

La $a, b \in G$

Vi har at $(a * b) * (a * b) = e$

$$a * (b * a) * b = e$$

$$a * a * (b * a) * b = a * e$$

$$e * (b * a) * b = a$$

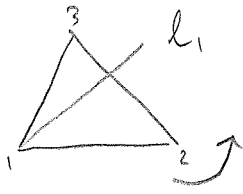
$$(b * a) * b * b = a * b$$

$$(b * a) * e = a * b$$

$$b * a = a * b$$

$\Rightarrow G$ er abelsk.

Extra 1:

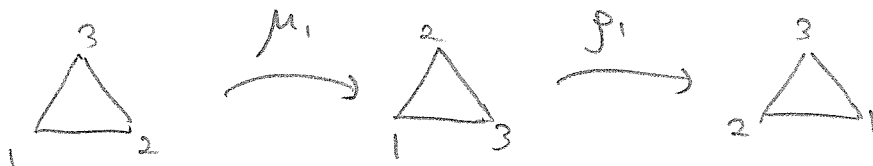
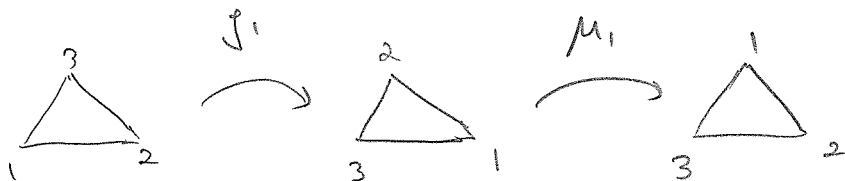


g_0 : Rotasjon 0°

g_1 : Rotasjon 120°

g_2 : Rotasjon 240°

μ_1 : Speiling om linja l_1

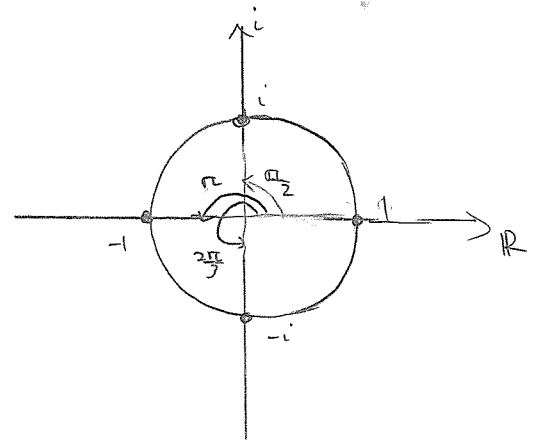


Seer at $\mu_1 \circ g_1 \neq g_1 \circ \mu_1$

Ekstra 2:

$$U_4 = (\{1, i, -1, -i\}, \cdot)$$

$$\mathbb{Z}_4 = (\{0, 1, 2, 3\}, +_4)$$



Ønsker en afbildning $\varphi: \mathbb{Z}_4 \rightarrow U_4$

$$\text{La } \varphi(0) = 1$$

$$\varphi(1) = i$$

$$\varphi(2) = -1$$

$$\varphi(3) = -i$$

Se fra tabellene under at dette giver en isomorfi $\mathbb{Z}_4 \cong U_4$

$+_4$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

$\cdot$	1	i	-1	-i
1	1	i	-1	-i
i	i	-1	-i	1
-1	-1	-i	1	i
-i	-i	1	i	-1