4.18.37

Let $a, b \in \mathcal{U}$. Then $b^{-1}a^{-1}ab = b^{-1}1b = b^{-1}b = 1$,

so $(ab)^{-1} = b^{-1}a^{-1}$, and $ab \in \mathcal{U}$, i.e. $\mathcal{U}$ is closed under multiplication.

$\mathcal{U}$ is a group:

i) $(ab)c = a(bc)$ from the fact that $\mathcal{U} \subset R$, and R is a ring

ii) $1 \in \mathcal{U}$, and is the identity under multiplication.

iii) From the definition of $\mathcal{U}$, every element has an inverse.

4.20.2

2 is a generator

$2 \cdot 2 = 4$
$4 \cdot 2 = 8$
$8 \cdot 2 = 16 \equiv 5 \mod 11$
$5 \cdot 2 = 10$
$10 \cdot 2 = 20 \equiv 9 \mod 11$
$9 \cdot 2 = 18 \equiv 7 \mod 11$
$7 \cdot 2 = 14 \equiv 3 \mod 11$
$3 \cdot 2 = 6$
$6 \cdot 2 = 12 \equiv 1 \mod 11$

4.20.4

$$3^{47} = (3^{22})^2 \cdot 3^3 \equiv 3^3 = 27 \equiv 4 \mod(23)$$

So the reminder when divided by 23 is 4

4.19.26

a) Anta at $ac = 0$ hvor $a \neq 0$ & $c \neq 0$

Veit at $\exists! b'$ s.a. $cb'c = c$.

$acb'ca = 0$ siden $ac = 0$

Veit at $\exists! b$ s.a. $aba = a$

Har $acb'ca + aba = a$

$a(cb'c + b)a = a$

Siden b er unik, så er $cb'c + b = b$

$\Rightarrow cb'c = 0$ $\lightning$ en selvmotsigelse.

$\Rightarrow R$ har ingen nulldivisorer.

b) Har $aba = a$, $a \neq 0$

$aba = ab$

$abab - ab = 0$

$a(bab - b) = 0$

R har ingen nulldivisorer

$\Rightarrow bab - b = 0$

$\Rightarrow \underline{b = bab}$ qed.

c) La $a \neq 0$. Veit at $\exists! \, b \in R$ s.a. $aba = a$

Ønsker å vise at ab er multiplikativ identitet, altså at

$$cab = c = abc \qquad \forall \, c \in R$$

Vi har at $caba = ca$

$$caba - ca = 0$$

$$(cab - c)a = 0 \qquad a \neq 0$$

$$\Rightarrow \quad cab - c = 0$$

$$cab = c$$

Andre siden er dualt, så $ab = 1$ i R.

d) Ser fra c) at for enhver $a \neq 0$ i R, så $\exists! \, b \in R$ slik at $ab = 1$.

$\Rightarrow$ R er en <u>divisjonsring</u>.