Norges teknisk–
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Institutt for matematiske fag

Contact during exam:
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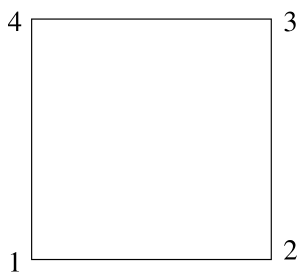
EKSAMEN I FAG MA2201 ALGEBRA

English
Fredag 28. mai 2004
Kl. 09.00 - 14.00
Sensur: 21. juni 2004

Permitted aids: HP30S

Problem 1

Let $G = \mathcal{D}_4$ be the symmetry group of the square



a) Write the 8 elements of G as permutations of $\{1, 2, 3, 4\}$.

b) How many ways can the 4 corners of the square above be painted when the colors yellow, blue and red are available? (Two colorings are viewed as equal if they can be carried over to each other by one of the symmetries of the square.)

Problem 2

Let G be the group of invertible 2×2-matrices over the real numbers $\mathbb{R}$. Let H be the subset of G consisting of matrices with determinant equal 1.

a) Show that H is a subgroup of G, and show that this subgroup is normal.

b) Show that the factor group G/H is isomorphic to the multiplicative group $\mathbb{R}^*$, i.e. $\mathbb{R} \setminus \{0\}$ where the group operation is the usual multiplication.

Problem 3

Let G be a group with 143 elements, and $H \subset G$ a subgroup for which $H \neq G$. Explain why H is a cyclic group.

Problem 4

a) Find all abelian groups of order 8 up to isomorphism.

b) Let G be the group of units in the commutative ring $\mathbb{Z}_{10} \times \mathbb{Z}_3$. Find all elements in G, and decide which of the groups in (a) G is isomorphic too.

Problem 5

Let $\sigma = \begin{pmatrix} 1\,2\,3\,4\,5\,6\,7\,8 \\ 3\,4\,5\,2\,1\,6\,7\,8 \end{pmatrix}$ be an element in S_8.

a) Write σ as a product of disjoint cycles and as a product of transpositions (i.e. cycles of length 2).

b) Find the order of σ? Find an element in S_8 of order 12.

Decide if there exists an element of order 27 in S_8? One of order 30?

Problem 6

La G være en gruppe med 12 elementer. Vis at G ikke er en simpel gruppe.