TMA4150/MA2201 - MIDTERM EXAM 2006

Tuesday, March 7, 2006 - **English**

Student number: ____________

The test consists of 10 multiple choice problems. In some of the problems, there are more than one correct answer. The number of correct answers should be clear from the text. Good luck!

Problem 1 Exactly **two** of these sets are groups with the given binary operation. Which?

- ☐ $\left\{ \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \mid a,b \in \mathbb{R},\ a^2+b^2 \neq 0 \right\}$ under ordinary matrix multiplication
- ☐ $\mathbb{R}^+$, the set of positive real numbers, under $*$, where $a*b=\sqrt{ab}$
- ☐ $\{1,2,3,4\}$ under multiplication modulo 5
- ☐ $\mathbb{R}^*$, the non-zero real numbers, under $*$, where $a*b=\dfrac{a}{b}$

Problem 2 How many non-isomorphic abelian groups are there of order 48?

☐ 1 ☐ 2 ☐ 4 ☐ 5 ☐ 8

Problem 3 What is the order of the element (3,4,5) in the group $\mathbb{Z}_{12}\times\mathbb{Z}_6\times\mathbb{Z}_{15}$?

☐ 3 ☐ 4 ☐ 5 ☐ 10 ☐ 12

Problem 4 Let

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 6 & 1 & 7 & 5 & 4 & 2 \end{pmatrix} \in S_7$$

Exactly **one** of the following expressions is equal to σ, written as a product of disjoint cycles. Which one?

- ☐ (13)(672)(64)
- ☐ (13)(2674)
- ☐ (6472)(13)
- ☐ (124)(367)
- ☐ (14)(32)(756)

Problem 5 Let

$$\tau = (1345) \in S_5$$

How many left cosets are there of the subgroup $\langle\tau\rangle \leq S_5$ generated by τ?

☐ 4 ☐ 10 ☐ 25 ☐ 30 ☐ 36

Problem 6 Let $G = \mathbb{Z}_{20}$, the cyclic group of order 20. How many non-isomorphic subgroups are there of G, not including G itself and the trivial subgroup?

☐ 2 ☐ 3 ☐ 4 ☐ 10

Problem 7 Exactly **two** of the following maps are group homomorphisms. Which?

☐ $\phi_1 : (\mathbb{R}, +) \to (\mathbb{R}^+, \cdot), \quad \phi_1(x) = e^x$

☐ $\phi_2 : (\mathbb{R}, +) \to (\mathbb{R}, +), \quad \phi_2(x) = |x|$

☐ $\phi_3 : S_n \to \mathbb{Z}_2, \quad \phi_3(\sigma) = \begin{cases} 0, & \sigma \text{ even} \\ 1, & \sigma \text{ odd} \end{cases}$

☐ $\phi_4 : (\mathbb{Z}, +) \to (\mathbb{Z}, +), \quad \phi_4(x) = x^2$

Problem 8 Exactly **two** of the following statements are correct. Which?

☐ If $\phi : G \to G'$ is a group homomorphism, and N is a normal subgroup of G, then $\phi[N]$ is a normal subgroup of G'.

☐ If G is an abelian group, and N is a normal subgroup of G, then the factor group G/N is an abelian group.

☐ If G is a non-abelian group, and N is a normal subgroup of G, then the factor group G/N is non-abelian.

☐ Every subgroup H of a group G with index $(G : H) = 2$ is a normal subgroup.

Problem 9 We want to paint the corners of an equilateral triangle. We have 5 different colours available, which can be used as many times as we want. Suppose for now that the corners are labelled. Let X be the set of possible colourings when the corners are labelled. How many elements of X stay the same when we reflect the triangle about the line l_1 which goes through one of the corners and is perpendicular to the opposite edge? (That is, find $|X_{\mu_1}|$ when μ_1 is reflection about the line l_1.)

☐ 5 ☐ 10 ☐ 20 ☐ 25 ☐ 30

Problem 10 In how many different ways can we paint the corners of the triangle in Problem 9, when the corners are not labelled? Two ways are considered to be equal if we can get one from the other by rotating and turning the triangle.

☐ 20 ☐ 25 ☐ 35 ☐ 90 ☐ 210

(Hint: If G is a finite group, and X is a finite G-set, then Burnside's Formula is given by

$$r \cdot |G| = \sum_{g \in G} |X_g|,$$

where r is the number of orbits.)