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TMA4150 Algebra and Number Theory

Wednesday May 25. 2005
Time: 9-13

Permitted aids: Calculator HP30S.

Gradings: June 10. 2005

1

a) Find all abelian groups with 8 elements, up to isomorphism.

b) Let G be the group of units in the commutative ring $\mathbb{Z}_{20}$. Write down all the elements in G, and decide which group in (a) G is isomorphic to.

2

a) Let $\sigma = (1\,2\,3\,4)\,(5\,6)\,(6\,8\,9)$ and $\tau = (1\,3\,6)(2\,4\,3\,5)$ be elements in the group S_{10} of permutations of $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Find the order of σ and $\sigma\tau$. Find an element of order 30 in S_{10}.

3

Show that the factor group $\mathbb{Z} \times \mathbb{Z}/\langle(1, 1)\rangle$ is isomorphic to the group $\mathbb{Z}$.

4

Let G be the group of invertible 2×2-matrices over $\mathbb{Z}_3$ under matrix multiplication. Let $H = \{\left(\begin{smallmatrix} a & o \\ b & c \end{smallmatrix}\right) \mid a, b, c \;\; i \;\; \mathbb{Z}_3, ac \neq 0\}$.

Show that H is a subgroup of G, but not a normal subgroup.

5

We want to make quadratic rugs of the form

1	2	3
4	5	6
7	8	9

where all 9 small squares are of equal size, and we shall color the rug on one side with black or white. In how many essentially different ways can this be done, when two coloring are viewed to be equal when one is obtained from the other by a rotation around the midpoint of the large square?

6

a) Find all zero divisors in the ring $\mathbb{Z}_{12}$.

b) Show that the commutative ring $\mathbb{R}[x]$ is not a field ($\mathbb{R}$ denotes the real numbers).

7 Let $p(x) = x^2 + 2x + 2$ be in $\mathbb{Z}_3[x]$.

a) Explain why the factor ring $F = \mathbb{Z}_3[x]/\langle p(x)\rangle$ is a field, and find the number of elements in F.

b) Show that $\alpha = x + \langle p(x)\rangle$ is a generator for the cyclic group $F \backslash \{0\}$ (under multiplication).