



Norwegian University of Science
and Technology
Institutt for matematiske fag

TMA4165 Differential
Equations and
Dynamical Systems
Spring 2018

Exercise set Challenge 1

This exercise set contains a challenging and interesting, but optional exercise!

1 Find a differentiable function $f : [0, \infty) \rightarrow \mathbb{R}$, which satisfies the integral equation

$$f^2(x) = \frac{1}{x} \int_1^x f(t) dt. \quad (1)$$

a) Show that f satisfies the differential equation

$$f\dot{f} = \frac{f - f^2}{2x}. \quad (2)$$

b) Show with the help of (2) that (1) has more than one solution.

c) Find a rigorous argument, based on the theory covered in the course, why the above integral equation may not have a unique solution.