Norwegian University of Science
and Technology
Institutt for matematiske fag

TMA4165 Differential Equations and Dynamical Systems Spring 2018

Exercise set 10

You find solutions to the following exercises on the web page. Give it a try and ask if something is unclear:
J.S.: 3.7, 3.12, 3.25, 8.3

These exercises will be supervised / discussed in the exercise class:

E28 Aim: Given the system

$$\dot{x} = f(x, y)$$
$$\dot{y} = g(x, y)$$

Show the following two criteria, which help to identify whether there exist periodic solutions or not.

a) Show that there are no closed paths in a simply connected region in which

$$\frac{\partial(\rho f)}{\partial x} + \frac{\partial(\rho g)}{\partial y}$$

is of one sign, where $\rho(x, y)$ is any function having continuous first partial derivatives. Deduce that

$$\dot{x} = -(1 - x)^3 + xy^2$$
$$\dot{y} = y + y^3$$

has no periodic solutions.

b) Show that there are no closed paths in a simply connected region in which

$$(\psi g)_x - (\psi f)_y = 0 \quad \text{and} \quad (f, g) \neq (0, 0)$$

where $\psi(x, y)$ is of one sign. Deduce that

$$\dot{x} = 2xy + x^3$$
$$\dot{y} = x^2 + y - y^2 + y^3,$$

has no periodic solutions.

E29 Aim: Show that the system

$$\begin{aligned} \dot{x} &= 1 - x^3 + y^2 \\ \dot{y} &= 2xy \end{aligned} \tag{1}$$

has no periodic solutions.

a) Show with the help of exercise E28 that the system (1) has no periodic solutions.

b) Find and classify all equilibrium points of the system (1). Use an argument based on the index of the equilibrium points to conclude that (1) has no periodic solutions.

E30 Aim: Determine whether or not the following systems have periodic solutions.

a) Show, with the help of index theory, that the system

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= 1 + x^2 - (1 - x)y \end{aligned}$$

has no periodic solutions.

b) Show that the system

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -x \\ \dot{z} &= 1 - x^2 - y^2 \end{aligned}$$

has no equilibrium points but nevertheless has periodic solutions.