Norwegian University of Science
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Institutt for matematiske fag

TMA4165 Differential Equations and Dynamical Systems Spring 2018

Exercise set 5

You find solutions to the following exercises on the web page. Give it a try and ask if something is unclear:
J. S.: 2.3 (ii), (iii), 2.29

Ex 2011.5 Given the system

$$\dot{x} = 2y(x^4 - 2x^2 + 2)$$
$$\dot{y} = 4(x - x^3)(y^2 + 1).$$

a) Show that the above system is Hamiltonian, and find a Hamiltonian function for the system.

b) Find and classify all equilibrium points of the above system.

Ex. 1999, 4 A dynamical system in polar coordinates is given by

$$\dot{\theta} = 1, \quad \dot{r} = \begin{cases} r^2 \sin(\frac{1}{r}) & \text{for } r > 0 \\ 0 & \text{for } r = 0. \end{cases} \tag{1}$$

Determine if the origin is a stable, asymptotically stable or unstable equilibrium point. Sketch the phase diagram nearby the origin.

These exercises will be supervised / discussed in the exercise class:

E13 Aim: Sketch the phase diagram for the following system

$$\dot{x} = y - x^4 + 1$$
$$\dot{y} = y + x^4 - 1.$$

a) Find and classify all equilibrium points of the above system.

b) Sketch the phase diagram, with orientation for the above system.

E14 Aim: Determine if the solutions of the following systems are Poincaré and / or Liapunov stable.

a) Show that all solutions of $\dot{\vec{x}} = A\vec{x} + \vec{v}$ have the same Liapunov stability as the zero solution to $\dot{\vec{x}} = A\vec{x}$, where A denotes some 2×2 matrix and $\vec{v} \in \mathbb{R}^2$.

b) Are the solutions of the system

$$\begin{aligned} \dot{x} &= 1, \\ \dot{y} &= 0 \end{aligned}$$

Poincaré and / or Liapunov stable?

c) Are the solutions of the system

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= 0 \end{aligned}$$

Poincaré and / or Liapunov stable.

d) Are the solutions of the system

$$\begin{aligned} \dot{x} &= x, \\ \dot{y} &= y \end{aligned}$$

Poincaré and / or Liapunov stable.

E15 Aim: Sketch the phase diagram for the following system

$$\begin{aligned} \dot{x} &= 2y(1 + e^{-x}) \\ \dot{y} &= e^{-x}(y^2 - 1). \end{aligned}$$

a) Determine whether or not the above system is Hamiltonian. If so find a Hamiltonian function.

b) Determine the two phase paths which separate the phase paths, which cross the x-axis, from the rest.

c) Sketch the phase diagram.