TMA4165 Differential Equations and Dynamical Systems Spring 2018

Exercise set 8

You find solutions to the following exercises on the web page. Give it a try and ask if something is unclear:
J.S.: 10.1 (i)–(iv), 10.2, 10.7

Exam 1996, 4 Given $V \in C^1(\mathbb{R}^n, \mathbb{R})$.

a) Show that, if $\mathbf{x_0}$ is a strict minimum for $V(\mathbf{x})$, then $\mathbf{x_0}$ is an asymptotically stable equilibrium point for the system

$$\dot{\mathbf{x}} = -\nabla V(\mathbf{x}).$$

b) Let

$$V(x, y) = x^2(x-1)^2 + y^2.$$

Sketch the phase diagram of the system

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = -\nabla V(x, y).$$

These exercises will be supervised / discussed in the exercise class:

E22 Aim: Show that all solutions to the system

$$\begin{aligned} \dot{x} &= x(y-b), \\ \dot{y} &= y(x-a), \end{aligned} \tag{1}$$

with a, $b > 0$, starting close to the origin approach the origin.

a) Find a strong Liapunov function at $(0, 0)$ for the system (1).

b) Confirm, with the help of the Liapunov function that all solutions starting in the domain $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 < 1$ approach the origin.

E23 Aim: Determine if the equilibrium point $(0, 0)$ of the following systems is asymptotically stable, (Liapunov) stable or unstable.

a)

$$\begin{aligned} \dot{x} &= -x^3 - 2xy \\ \dot{y} &= x^2 - 3y^3. \end{aligned}$$

b)

$$\dot{x} = -x - x^2 y$$
$$\dot{y} = 2x^2 + y.$$

$\boxed{\text{E24}}$ Aim: Given the system

$$\begin{aligned}\dot{x} &= 2(x^2 + 2y^2)y - x^3 \\ \dot{y} &= -(x^2 + 2y^2)x - e^x y.\end{aligned} \tag{2}$$

Show that the domain of attraction of $(0, 0)$ is all of $\mathbb{R}^2$.

a) Show that $(0, 0)$ is an asymptotically stable equilibrium point for the system (2).

b) Show that all solutions to (2) approach the origin.