Norwegian University of Science and Technology
Institutt for matematiske fag

TMA4165 Differential Equations and Dynamical Systems
Spring 2018

Exercise set Challenge 2

This exercise set contains a challenging and interesting, but optional exercise!

1 Prove the following theorem: The equation

$$\ddot{x} + f(x)\dot{x} + g(x) = 0$$

has no periodic solution if

- f and g are Lipschitz continuous
- $F(x) = \int_0^x f(u)du$ is an odd function with $F'(0) > 0$
- $F(x)$ is zero only at $x = -a$, $x = 0$, $x = a$ for some $a > 0$
- $F(x) \to \infty$ as $x \to \infty$ monotonically for $x > a$
- $g(x)$ is an odd function, and $g(x) > 0$ for $x > 0$.

In particular, all solutions tend to $(0, 0)$ as $t \to \infty$.

Hint: The proof follows closely the one of Theorem 11.4 in Jordan and Smith, where it is proven that
The equation

$$\ddot{x} + f(x)\dot{x} + g(x) = 0$$

has a unique periodic solution if

- *$f(x)$ and $g(x)$ are Lipschitz continuous*
- *$F(x) = \int_0^x f(u)du$ is an odd function with $F'(0) < 0$*
- *$F(x)$ is zero only at $x = -a$, $x = 0$, $x = a$ for some $a > 0$*
- *$F(x) \to \infty$ $(x \to \infty)$ monotonically for $x > a$*
- *$g(x)$ is odd and $g(x) > 0$ when $x > 0$*