

$\psi_1(t), \psi_2(t), \dots, \psi_n(t) : \mathbb{R} \rightarrow \mathbb{R}^n$ CONT

$\psi_i(t) \neq 0 \quad \forall t$



$\exists \alpha_1, \dots, \alpha_n$ NOT ALL ZERO

ST $\alpha_1 \psi_1(t) + \alpha_2 \psi_2(t) + \dots + \alpha_n \psi_n(t) = 0 \quad \forall t$

$\psi_1(t), \psi_2(t), \dots, \psi_n(t)$ LIN DEP

$$\dot{\vec{x}}(t) = A(t)\vec{x}(t)$$

($A(t)$... $n \times n$ MATRIX
ENTRIES FUNCTIONS OF t)

$n+1$ SOLUTIONS
 $\psi_1(t), \dots, \psi_{n+1}(t)$

LIN. DEP

$\psi_1(t_0), \psi_2(t_0), \dots, \psi_{n+1}(t_0)$
LIN DEP $\forall t_0$.

$\exists n$ LIN IND SOLUTIONS.

$\psi_1(t), \dots, \psi_n(t)$

$\psi_1(t_0), \dots, \psi_n(t_0)$
LIN IND $\forall t_0$

$\phi(t)$ SOLUTION TO $\dot{\vec{x}}(t) = A(t)\vec{x}(t)$

$\exists \alpha_1, \alpha_2, \dots, \alpha_n$ ST
 $\phi(t) = \alpha_1 \psi_1(t) + \alpha_2 \psi_2(t) + \dots + \alpha_n \psi_n(t)$

$$\dot{\vec{x}} = A(t)\vec{x}$$

$(A(t)) \dots n \times n$ MATRIX
ENTRIES FUNCTIONS OF t



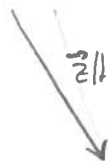
$\exists n$ LIN IND SOLUTIONS
 $\phi_1(t), \dots, \phi_n(t)$



$\Phi(t) = [\phi_1(t), \phi_2(t), \dots, \phi_n(t)]$.
FUNDAMENTAL MATRIX



$$\det(\Phi(t)) \neq 0 \quad \forall t$$



$\vec{z}(t)$ SOLUTION WITH $\vec{z}(t_0) = \vec{z}_0$

$$\vec{z}(t) = \Phi(t)\Phi^{-1}(t_0)\vec{z}_0.$$