$\psi_1(t), \psi_2(t), \dots, \psi_n(t) : \mathbb{R} \to \mathbb{R}^m$ CONT

$\psi_i(t) \not\equiv 0 \quad \forall i$

$\Downarrow$ $\exists \alpha_1, \dots, \alpha_n$ NOT ALL ZERO ST $\alpha_1\psi_1(t) + \alpha_2\psi_2(t) + \dots + \alpha_n\psi_n(t) = 0 \quad \forall t$

$\psi_1(t), \psi_2(t), \dots, \psi_n(t)$ LIN DEP

$\dot{\vec{x}}(t) = A(t)\vec{x}(t)$

($A(t)$.. $n \times n$ MATRIX ENTRIES FUNCTIONS OF t)

$n+1$ SOLUTIONS $\psi_1(t), \dots, \psi_{n+1}(t)$ $\rightarrow$ LIN. DEP

$\downarrow$

$\psi_1(t_0), \psi_2(t_0), \dots, \psi_{n+1}(t_0)$ LIN DEP $\forall t_0$.

$\rightarrow$ $\exists n$ LIN IND SOLUTIONS. $\psi_1(t), \dots, \psi_n(t)$

$\downarrow$

$\psi_1(t_0), \dots, \psi_n(t_0)$ LIN IND $\forall t_0$

$\downarrow$ $\phi(t)$ SOLUTION TO $\dot{\vec{x}}(t) = A(t)\vec{x}(t)$

$\exists \alpha_1, \alpha_2, \dots, \alpha_n$ ST $\phi(t) = \alpha_1\psi_1(t) + \alpha_2\psi_2(t) + \dots + \alpha_n\psi_n(t)$.

$$\dot{\vec{x}} = A(t)\vec{x}$$

($A(t)$... $n \times n$ MATRIX
ENTRIES FUNCTIONS OF t)

$$\Downarrow$$

$\exists n$ LIN IND SOLUTIONS
$\phi_1(t), \dots, \phi_n(t)$

$$\Downarrow$$

$\Phi(t) = [\phi_1(t), \phi_2(t), \dots, \phi_n(t)]$.
FUNDAMENTAL MATRIX

$\swarrow$ $\qquad$ $\searrow$ $\vec{z}(t)$ SOLUTION WITH $\vec{z}(t_0) = \vec{z}_0$

$\det(\Phi(t)) \neq 0 \ \forall t$

$\vec{z}(t) = \Phi(t)\Phi^{-1}(t_0)\vec{z}_0$.