

$$\begin{aligned}\dot{x} &= f(x,y) \\ \dot{y} &= g(x,y)\end{aligned}$$

$(0,0)$... EQUILIBRIUM POINT

$V: \mathcal{N} \rightarrow \mathbb{R}$, $\mathcal{N}$... NEIGHBORHOOD OF $(0,0)$

S.T. V, V_x, V_y CONT

$V(x,y) > 0$ FOR $(x,y) \neq (0,0)$

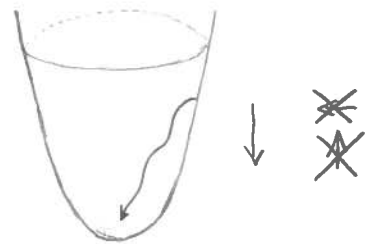
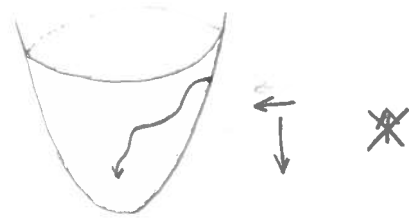
$V(0,0) = 0$

$$V_x \dot{x} + V_y \dot{y} \leq 0 \quad \forall (x,y) \in \mathcal{N}$$

$\vec{x}(t) = \vec{0}$... LIAPUNOV STABLE

$$V_x \dot{x} + V_y \dot{y} < 0 \quad \forall (x,y) \in \mathcal{N} \setminus \{0\}$$

$\vec{x}(t) = \vec{0}$... ASYMPTOTICALLY STABLE



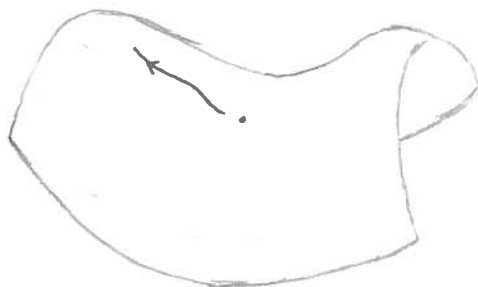
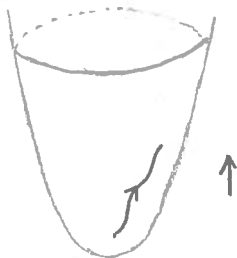
$$\begin{aligned}\dot{x} &= f(x, y) \\ \dot{y} &= g(x, y) \\ (0,0) &\text{ EQUILIBRIUM POINT}\end{aligned}$$

$U \subset \mathbb{R}^2$, N NEIGHBORHOOD OF $(0,0)$

$$\begin{aligned}U, f, g &\text{ CONT.} \\ U(0,0) &= 0.\end{aligned}$$

$$\begin{aligned}\dot{U}(0,0) &= 0 \\ \dot{U}(x,y) &= U_x f + U_y g(x,y) > 0 \\ \forall \delta > 0 \exists \vec{x}_\delta \in B_\delta(\vec{0}) \text{ ST } U(\vec{x}_\delta) > 0\end{aligned}$$

$\vec{x}(t) = 0$ UNSTABLE



$\forall \epsilon > 0$ (SMALL ENOUGH) $\forall 0 < \delta < \epsilon$
 $\exists \vec{x}_\delta \in B_\delta(\vec{0})$ ST $\vec{x}_\delta(t)$ LEAVES
 $B_\epsilon(\vec{0})$