$\dot{x} = f(x,y)$

$\dot{y} = g(x,y)$

$(0,0)$... EQUILIBRIUMPOINT

$V: \mathcal{N} \to \mathbb{R}$, $\mathcal{N}$... NEIGHBORHOOD OF $(0,0)$

ST V, V_x, V_y CONT

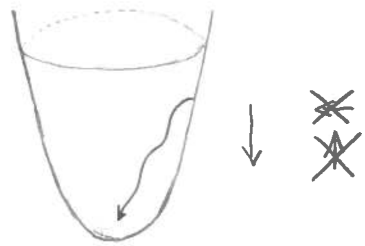
$V(x,y) > 0$ FOR $(x,y) \neq (0,0)$

$V(0,0) = 0$

$$\Big\downarrow \; V_x\dot{x} + V_y\dot{y} \leq 0 \quad \forall (x,y) \in \mathcal{N}$$

$\vec{x}(t) = \vec{0}$... LIAPUNOV STABLE

$$\Big\downarrow \; V_x\dot{x} + V_y\dot{y} < 0 \quad \forall (x,y) \in \mathcal{N} \setminus \{0\}$$



$\vec{x}(t) = \vec{0}$... ASYMPTOTICALLY STABLE.

$\dot{x} = f(x,y)$

$\dot{y} = g(x,y)$

$(0,0)$... EQUILIBRIUMPOINT

$U: \mathcal{N} \to \mathbb{R}$, $\mathcal{N}$... NEIGHBORHOOD OF $(0,0)$

U, U_x, U_y.. CONT

$U(0,0) = 0$.

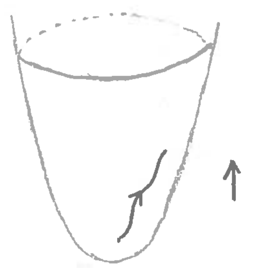
$\dot{U}(0,0) = 0$

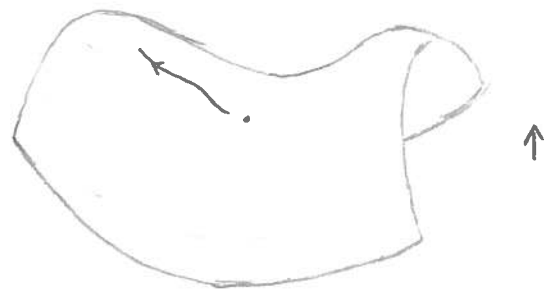
$\dot{U}(x,y) = U_x f + U_y g(x,y) > 0$.

$\forall \delta > 0 \ \exists \vec{x}_\delta \in B_\delta(\vec{0})$ ST $U(\vec{x}_\delta) > 0$

$\downarrow$

$\vec{x}(t) = 0$ ... UNSTABLE





$\forall \varepsilon > 0$ (SMALL ENOUGH) $\forall 0 < \delta < \varepsilon$

$\exists \vec{x}_\delta \in B_\delta(\vec{0})$ ST $\vec{x}_\delta(t)$ LEAVES

$B_\varepsilon(\vec{0})$.